

Deep Reinforcement Learning

Building Blocks

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 @boelger

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<https://join.slack.com/t/deep-rl-tutorial/signup>

The Plan

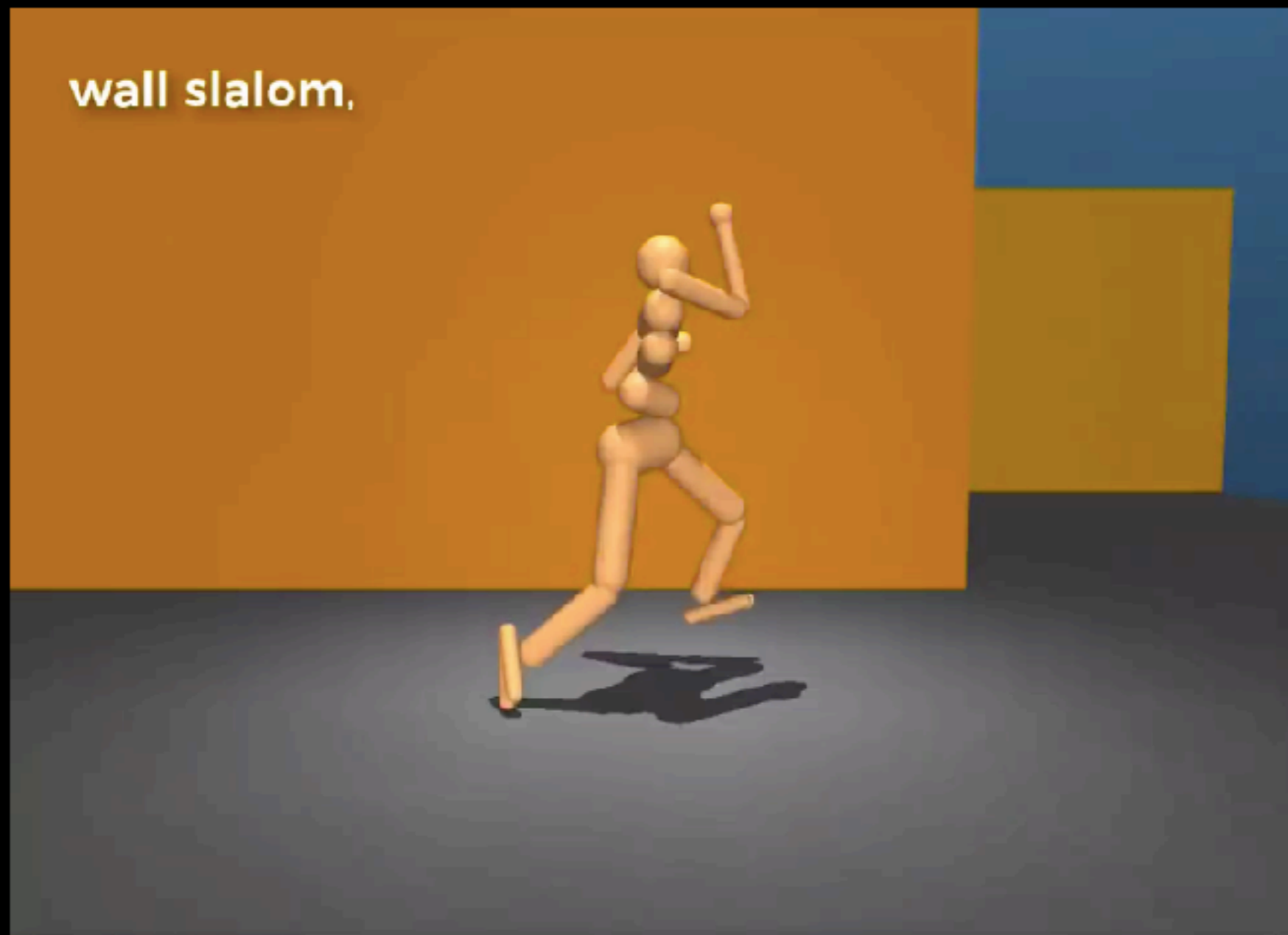
- The Problem
- (deep) RL Concepts by Example
- Problem Decomposition
- Solution Methods
 - Value Based
 - Policy Based
 - Actor-Critic

how to make
decisions over time
to maximise my
return / “long term reward”?

<http://cs.stanford.edu/groups/littledog/>



emergence of locomotion



<https://deepmind.com/blog/producing-flexible-behaviours-simulated-environments/>

https://www.youtube.com/watch?v=hx_bgoTF7bs

<https://arxiv.org/abs/1707.02286>

Rich Sutton et al.



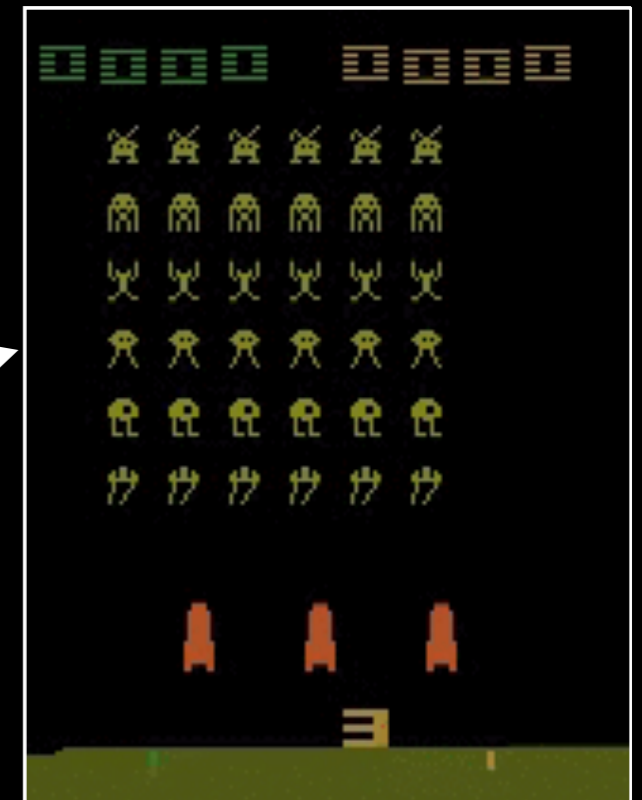
As we know...

Stanford



<http://heli.stanford.edu/>

Vlad Mnih et. al.



late
1980s

RL for robots using
NNs, L-J Lin. **PhD**
1993, CMU

Gerald Tesauro



1995

2004

Google DeepMind



David Silver et. al.



2015 —

2013 —



Problem Characteristics

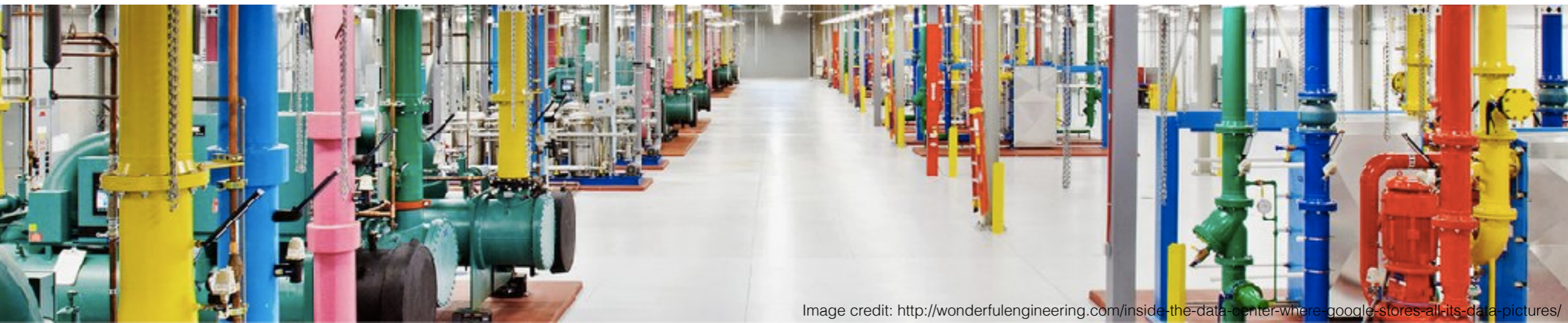
dynamic

uncertainty/volatility

uncharted/**unimagined**/
exception laden

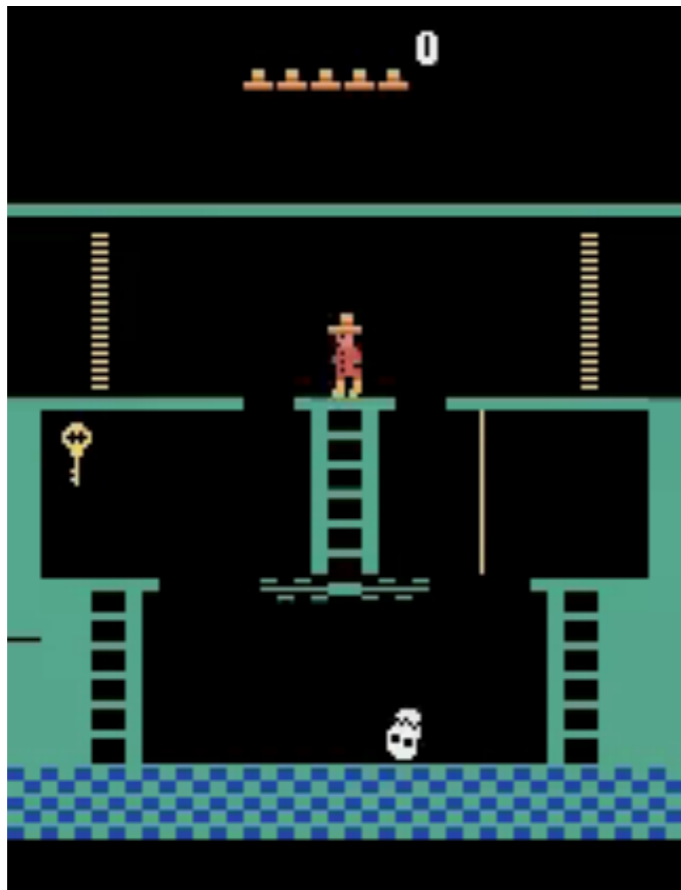
delayed consequences

requires **strategy**



Solution

machine with **agency** which **learn**, **plan**, and **act** to find a strategy for solving the problem



autonomous to some extent

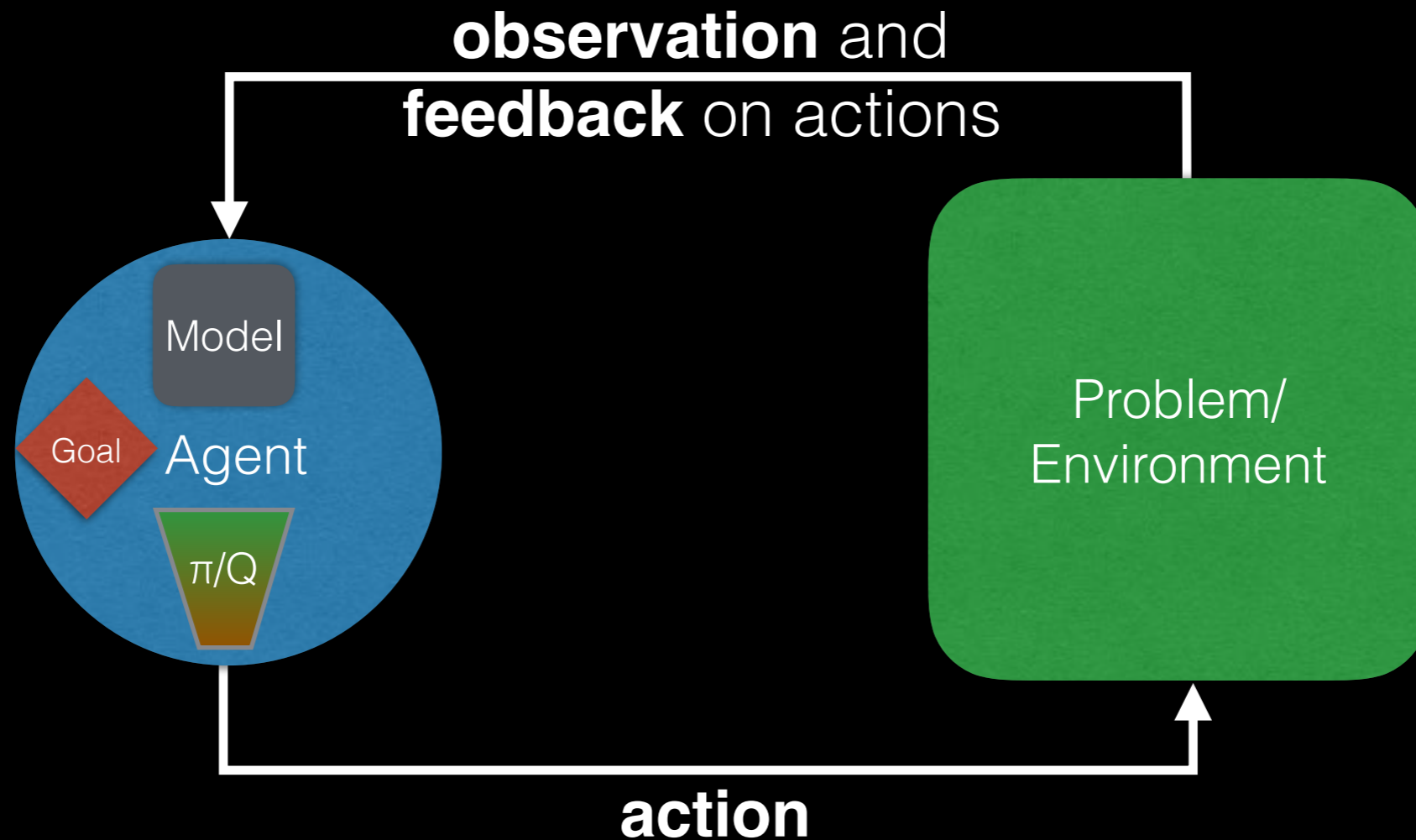
probe and **learn from feedback**

focus on the **long-term objective**

explore and **exploit**

what is the
sequence of actions
I could take to maximise my
return / “long term reward”?

Reinforcement Learning

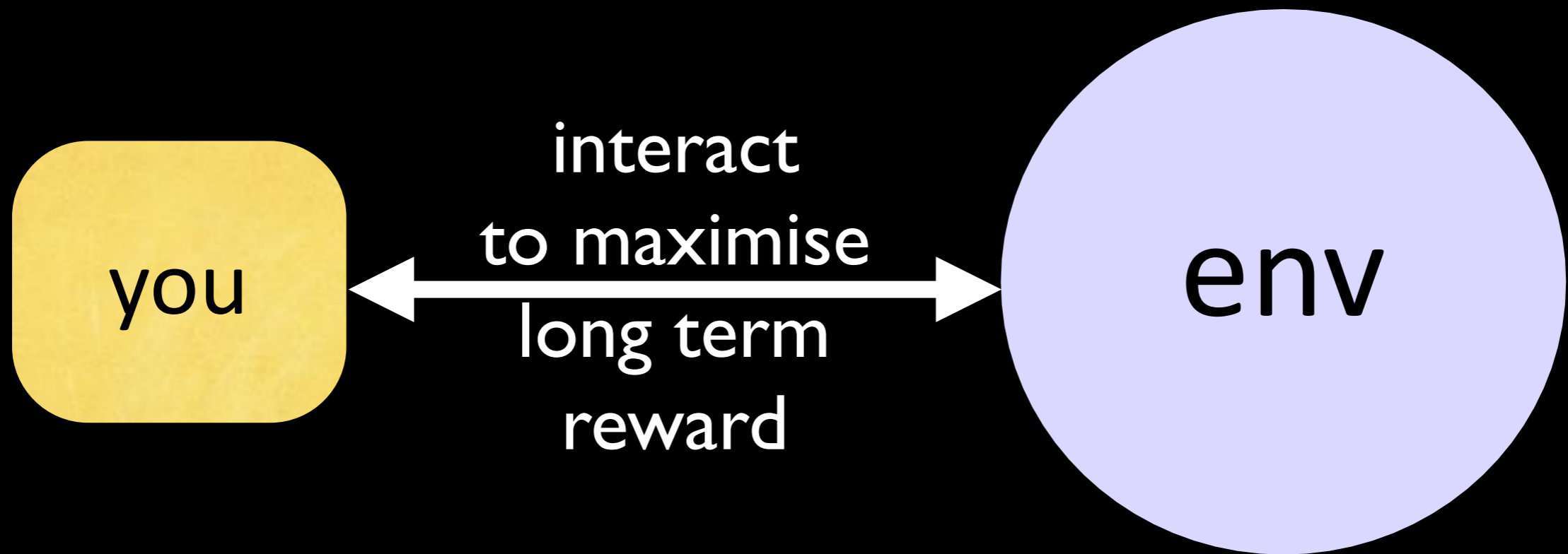


 Goal maximise return $\mathbf{E}\{R\}$

 Model dynamics model

 π/Q policy/value function

the excruciatingly
awesome MDP game!

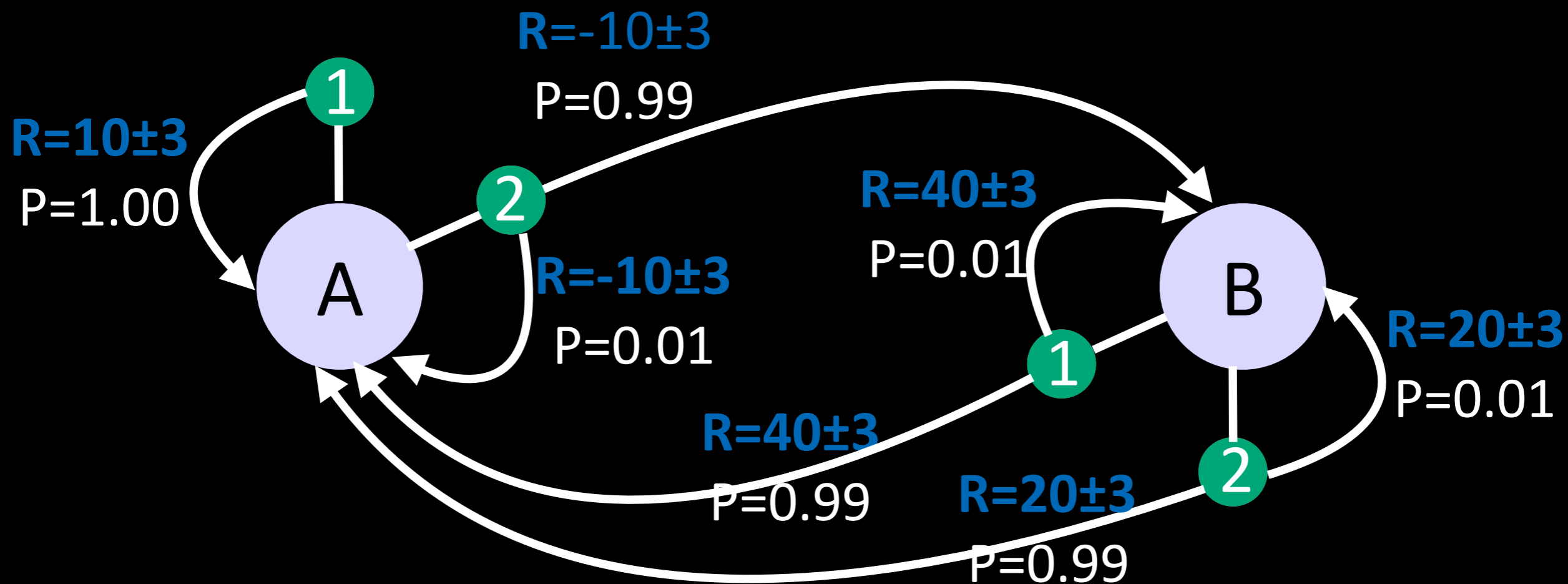


Inspired by Rich Sutton's tutorial:

<https://www.youtube.com/watch?v=ggqnxyjaKe4>

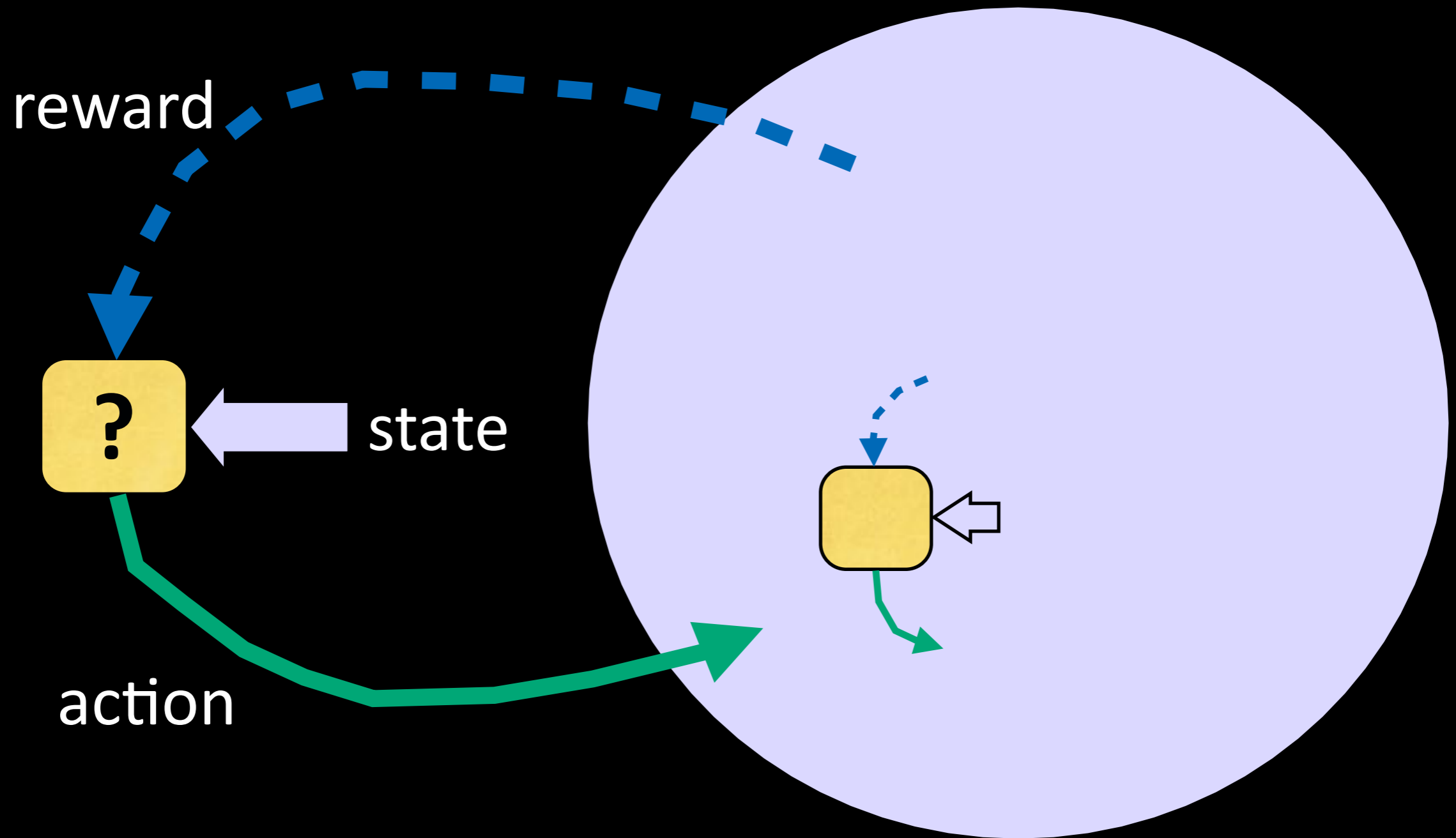
the MDP (S, A, P, R, γ)

R: immediate reward function $R(s, a)$
P: state transition probability $P(s' | s, a)$

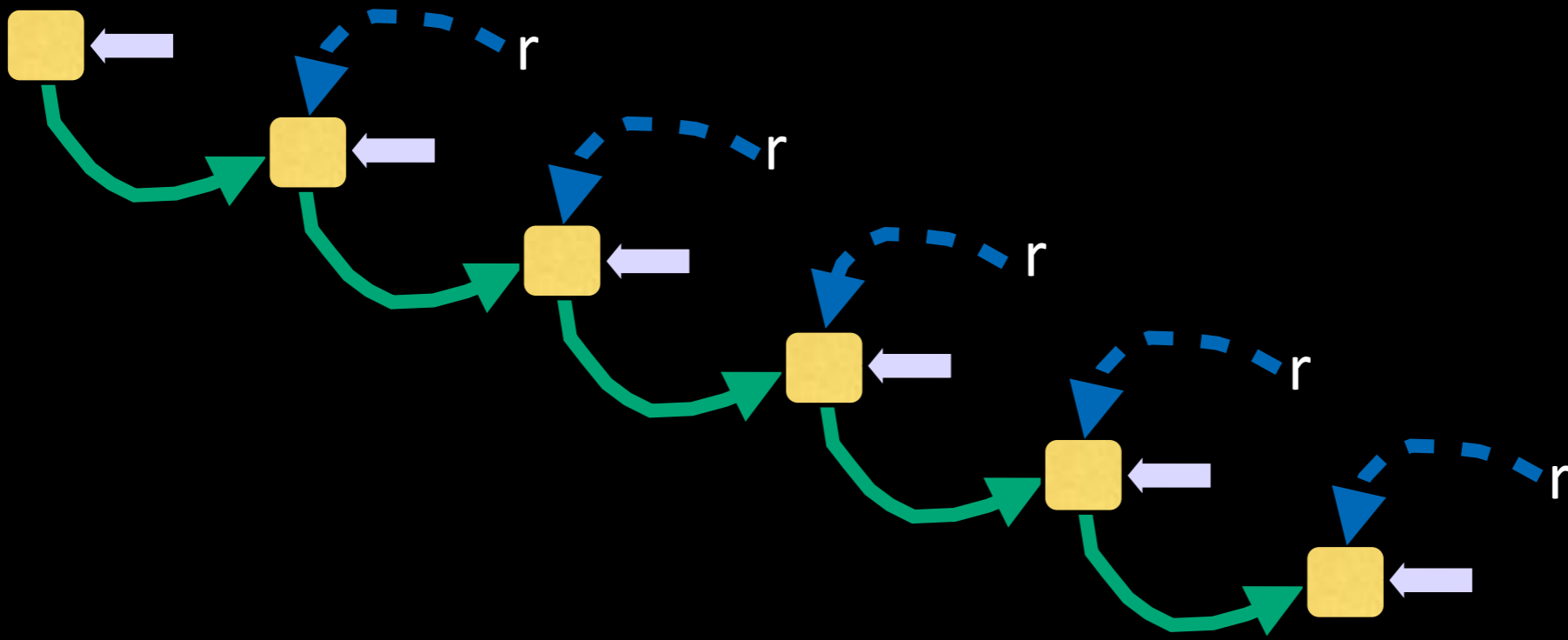


<https://github.com/traai/basic-rl>

the problem (cartoon of an MPD)



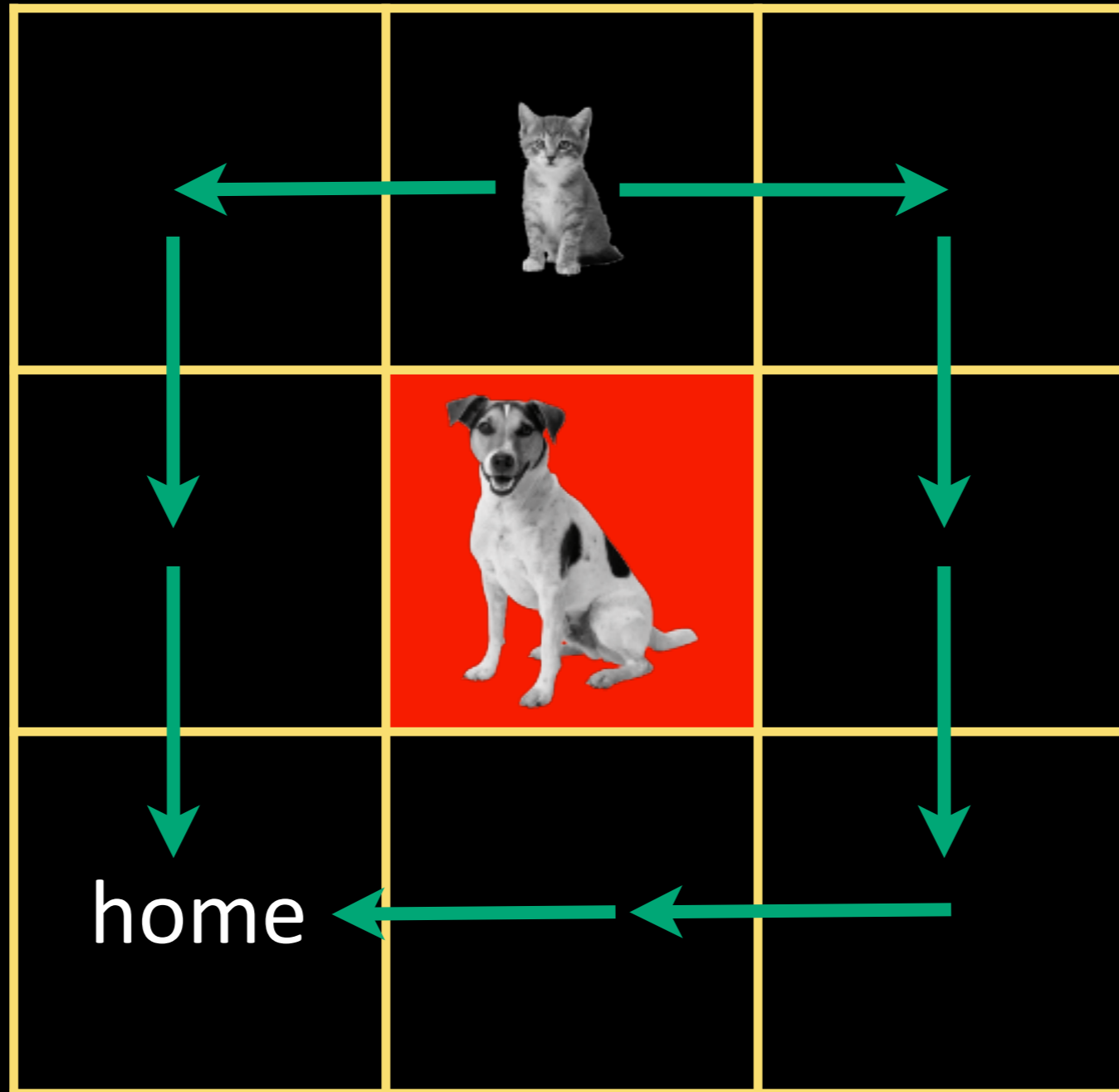
agent's job/goal?



maximise **expected**

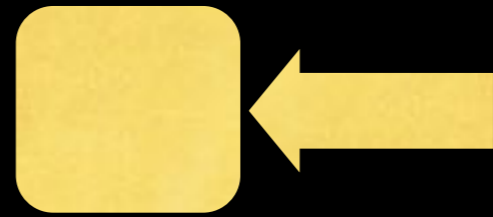
cumulative reward/return

toy problem



state and action spaces

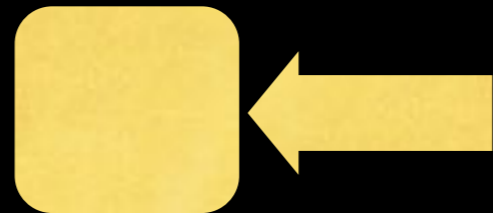
- size of these spaces can be **quite large**
- specifying the spaces is crucial in designing a good learning agent



5 integer values between
1 and 100: {22,44,12,67,9}

size of state space = $100 \times 100 \times 100 \times 100 \times 100$

can quantise state space differently

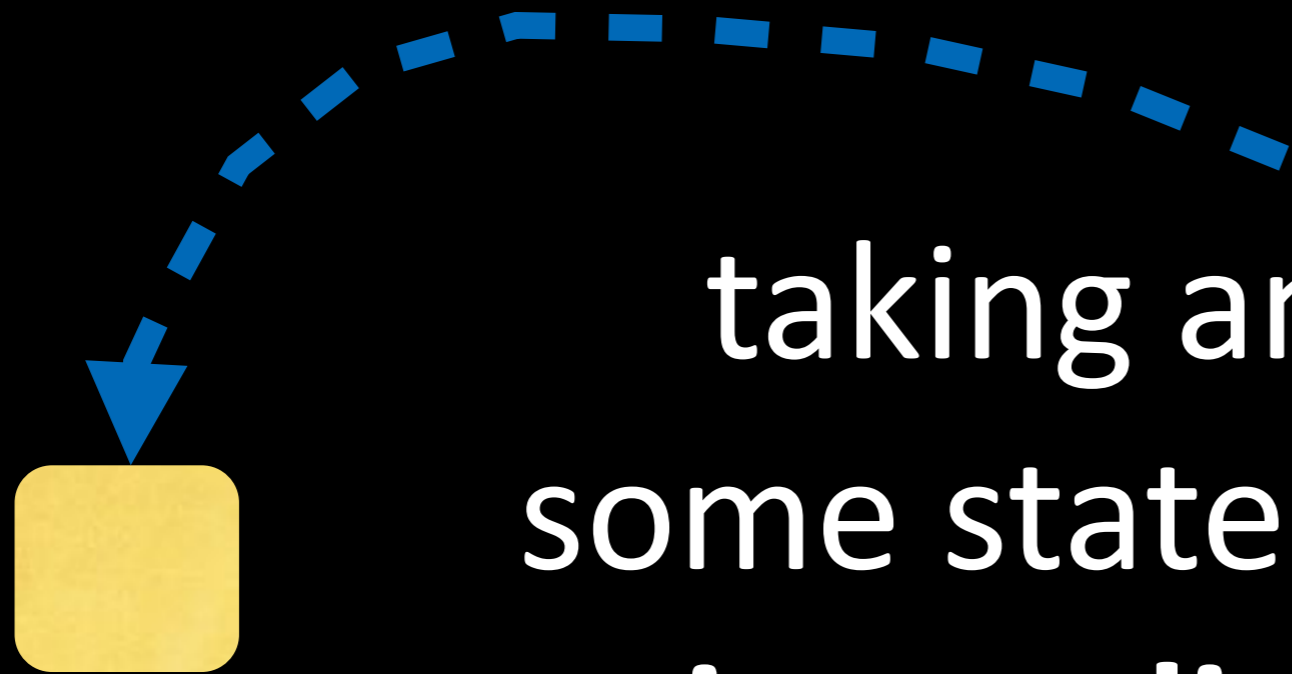


5 values belonging
to 2 classes: {1, 2, 1, 2, 1}

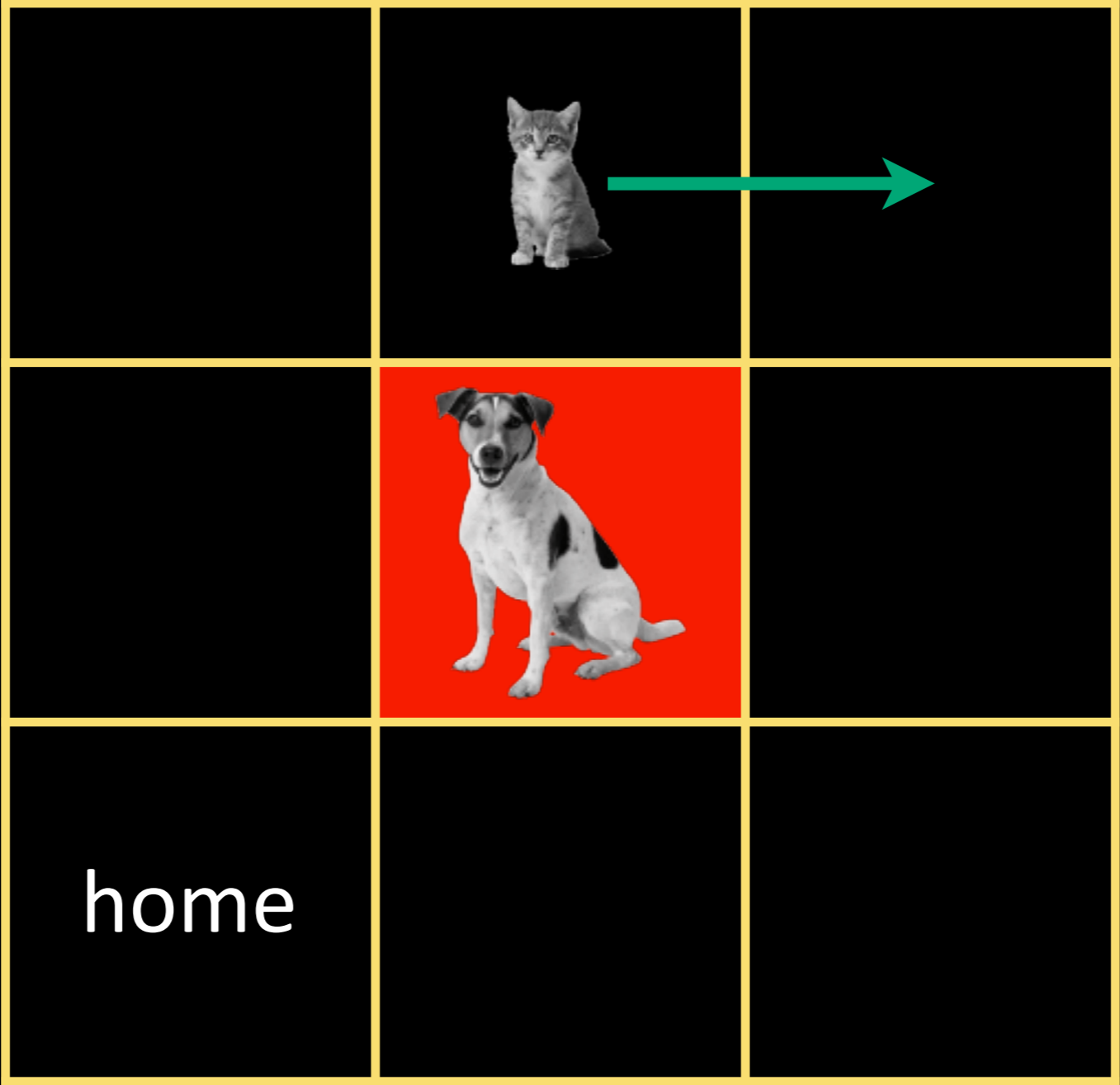
size of state space = $2 \times 2 \times 2 \times 2 \times 2$

in the toy problem? 9

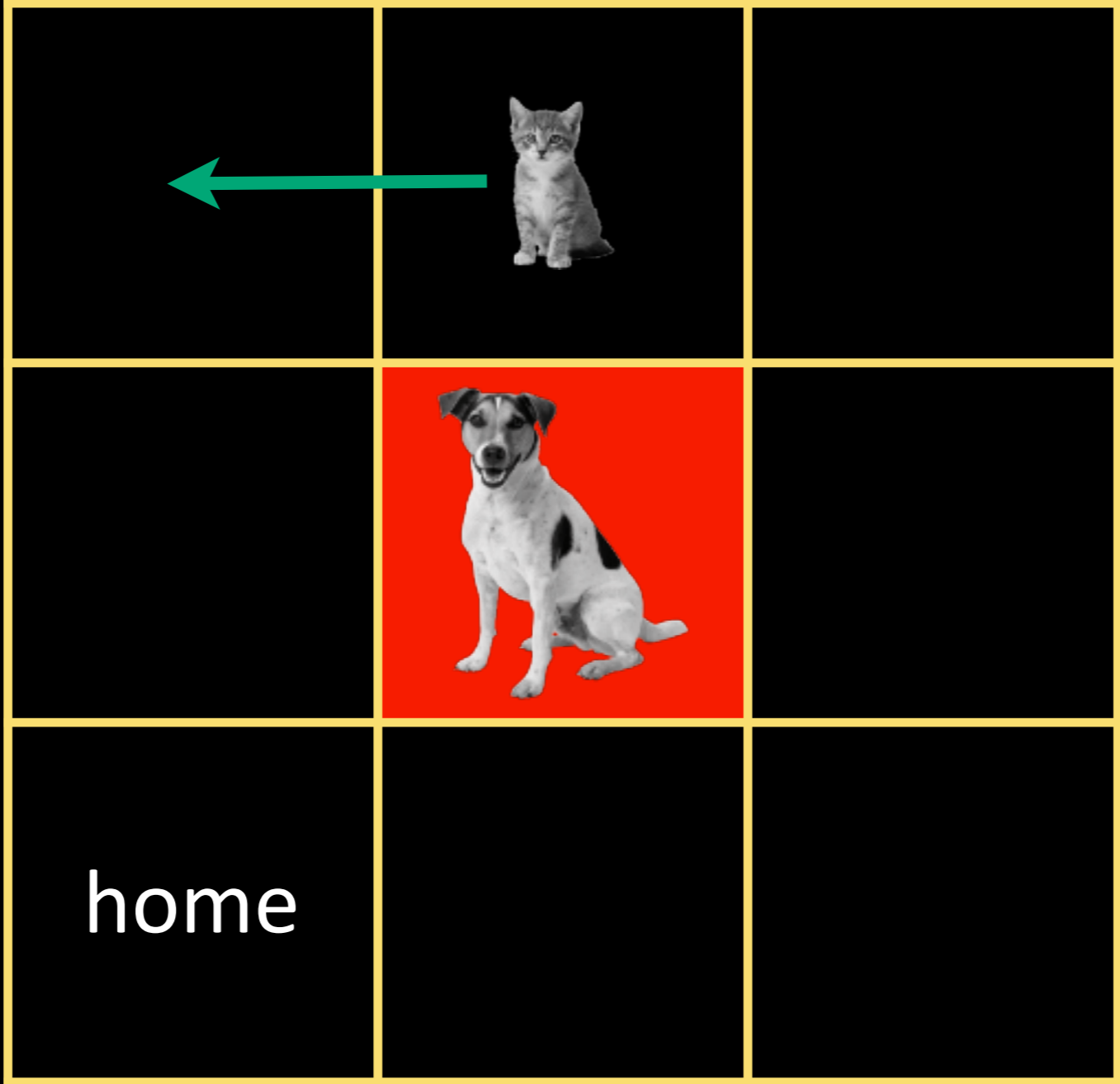
reward

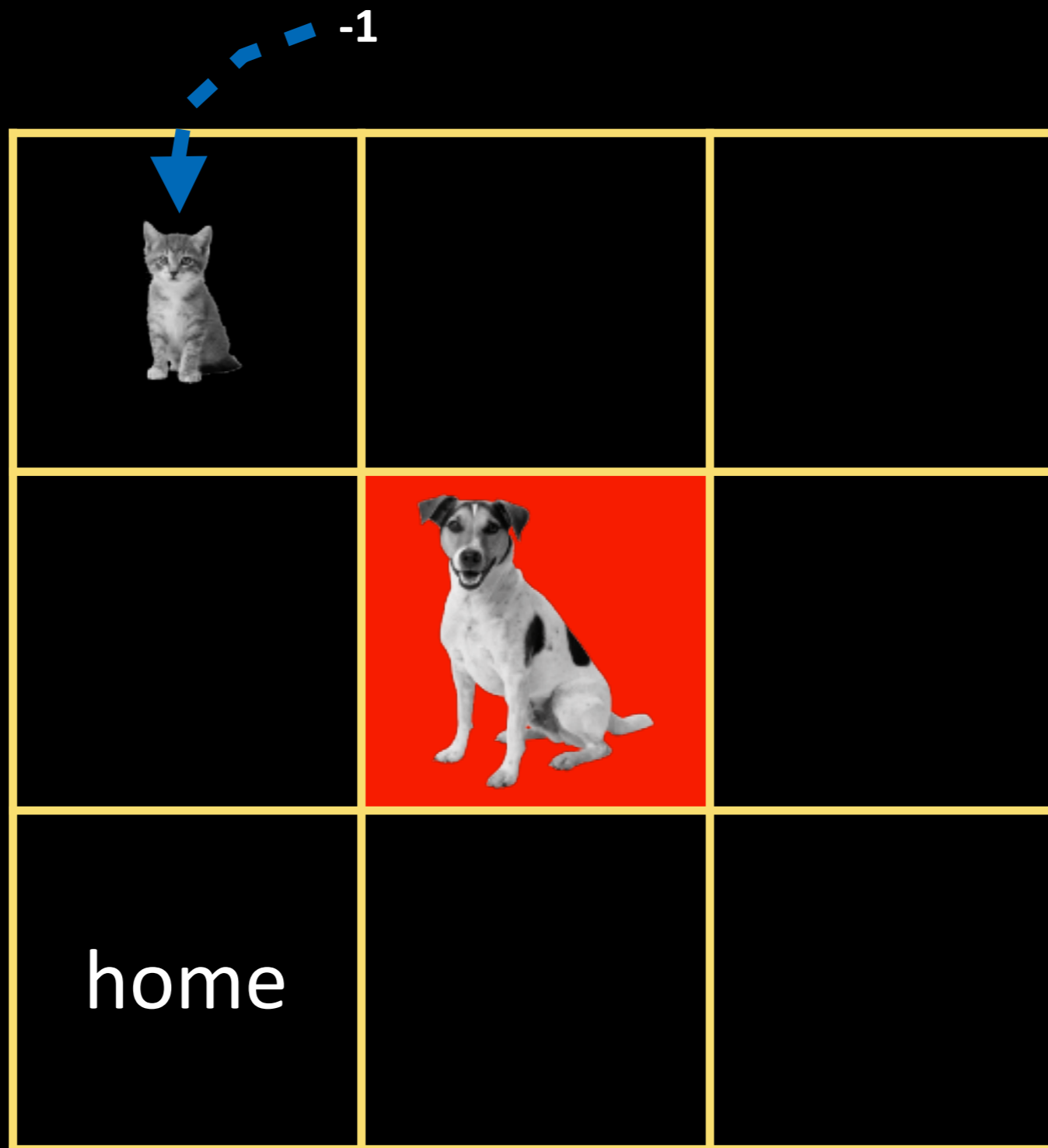


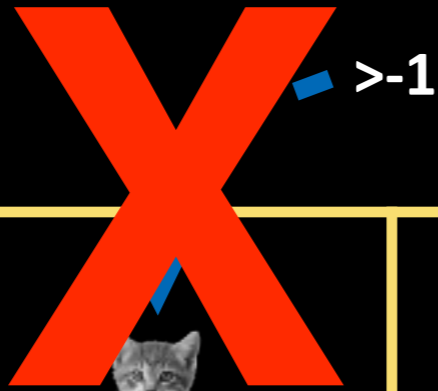
taking an action in
some state results in an
immediate reward
(can be negative)













		
		
home		what if the cat were to start here some other day?

reward system should tell
the agent:

what to achieve

rather than how to achieve

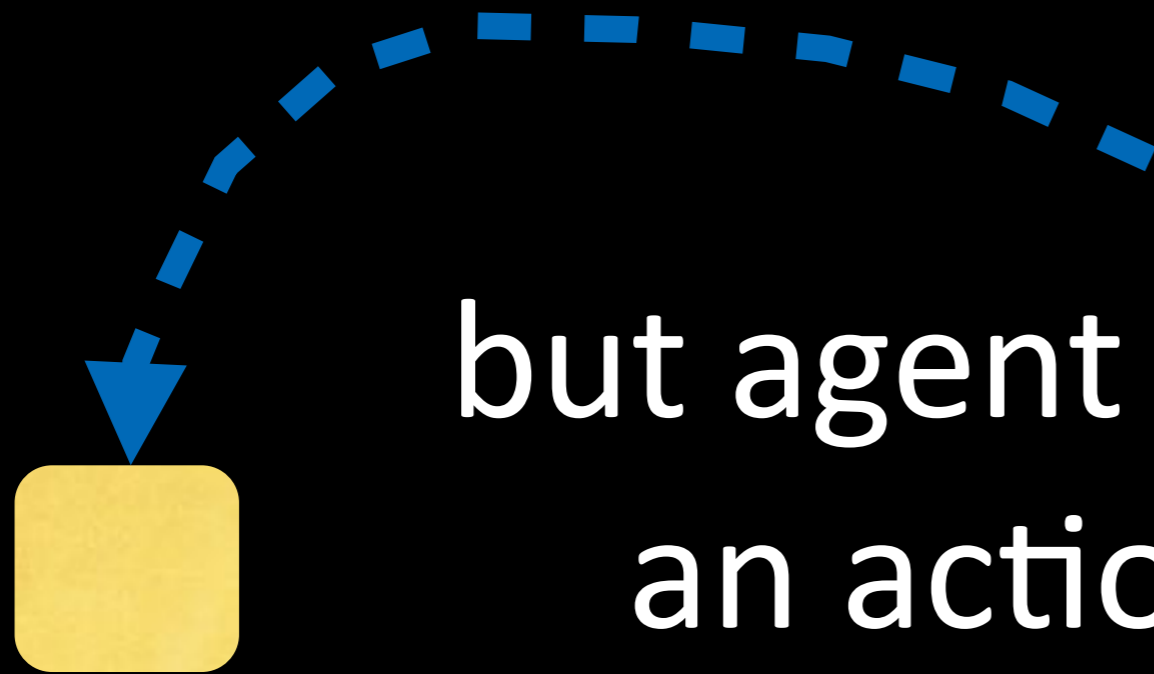
reward



this is all the
feedback an agent gets!

immediate!

reward



but agent has to choose
an action based on
expected return

expected return



task

```
graph TD; task --> episodic; task --> continual;
```

episodic

(there is an **end**)

continual

(there is no **end**)

episodic

(there is an **end**)

agent taking **finite (say 5) steps** till the end...

should act based on the

e.g. average of the following

$$R_0 = r_1 + r_2 + r_3 + r_4 + r_5$$

continual
(there is no **end**)

agent can continue acting for **infinite steps**
in time...

should **discount** future rewards and act based on
e.g. average of the following

$$R_0 = r_1 + \gamma r_2 + \gamma^2 r_3 + \gamma^3 r_4 + \gamma^4 r_5 + \dots$$

discount

future reward is probably more
uncertain than immediate reward

shortsighted?

$$\gamma = 0$$

$$0 \leq \gamma \leq 1$$

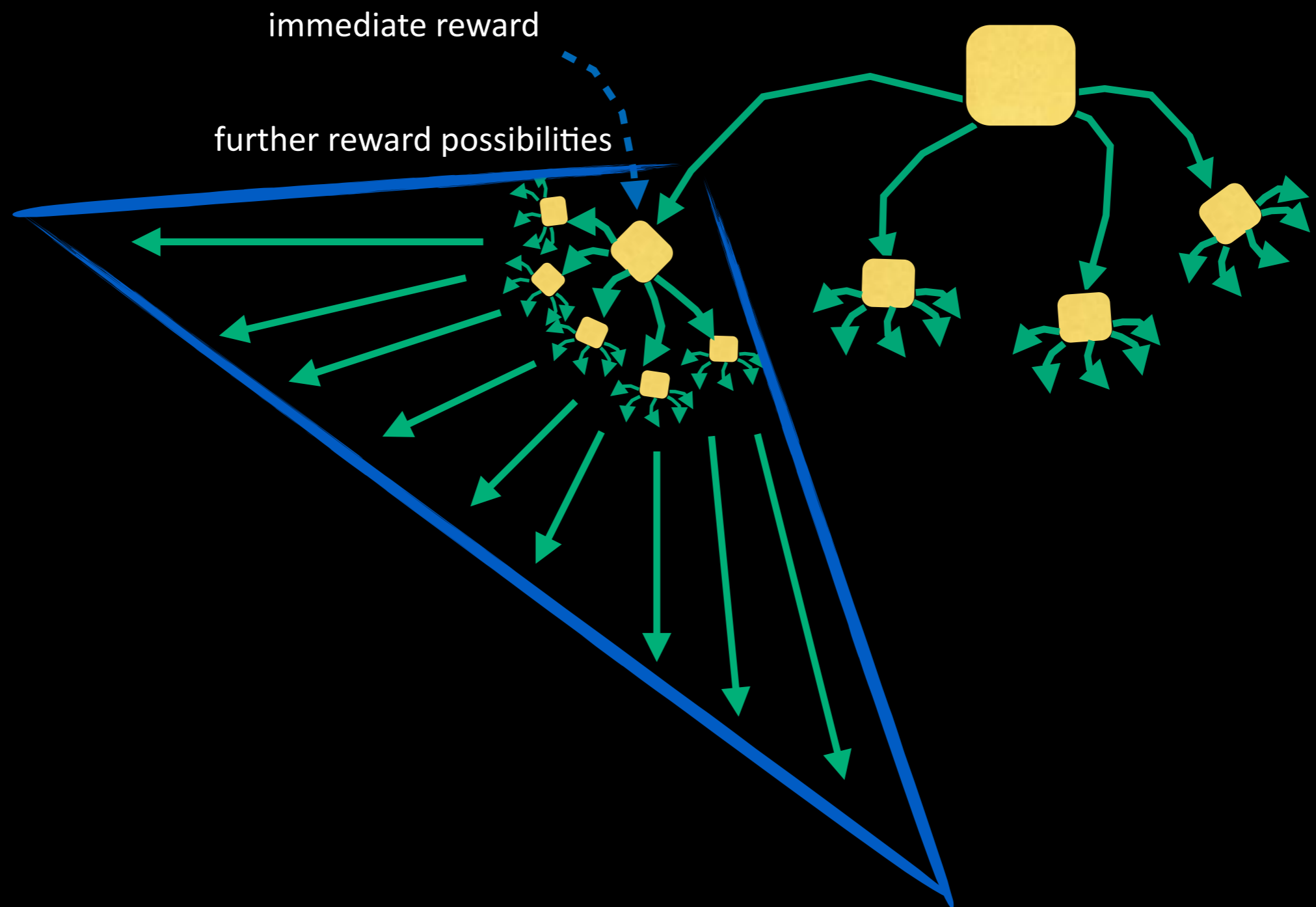
farsighted?

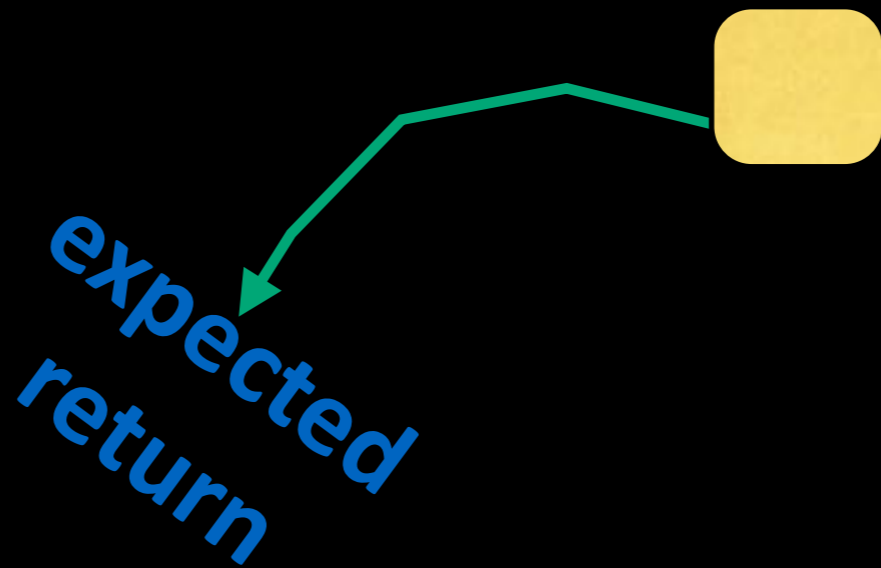
$$\gamma = 1$$

$$R_0 = r_1 + \gamma r_2 + \gamma^2 r_3 + \gamma^3 r_4 + \gamma^4 r_5 + \dots$$

$$R_0 = \sum_{k=0}^T \gamma^k r_{k+1}$$

$$E \left\{ R_t = \sum_{k=0}^T \gamma^k r_{t+k+1} \right\}$$





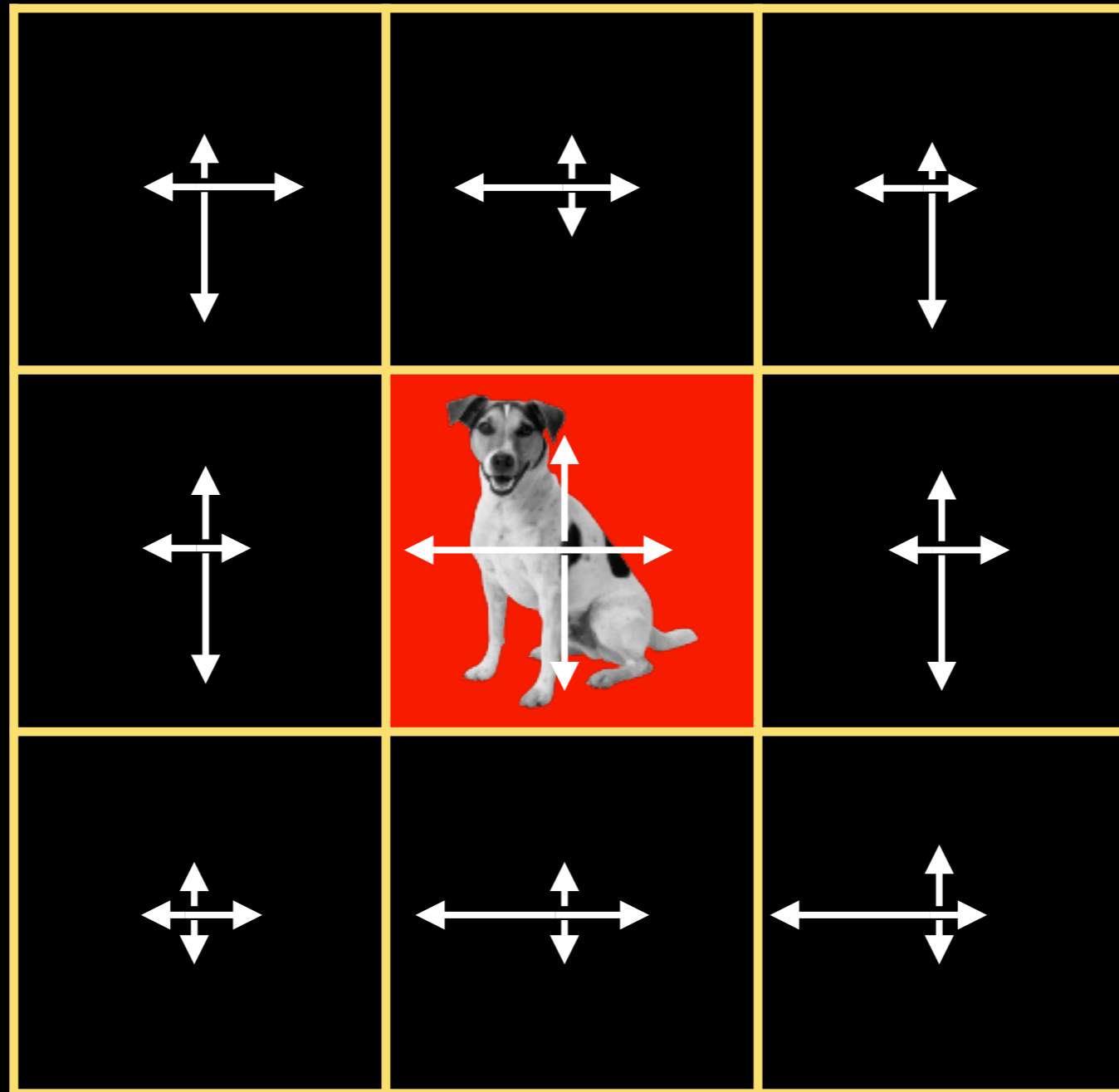
$$E \left\{ R_t = \sum_{k=0}^T \gamma^k r_{t+k+1} \right\}$$

but these expected
returns are
not known to agent
beforehand!

what knowledge might
the agent try to acquire
to behave properly?




ranking/probability of an action in some state
bringing max expected return (long term value)?



expected long term value of
being in each state,
under some action selection scheme?



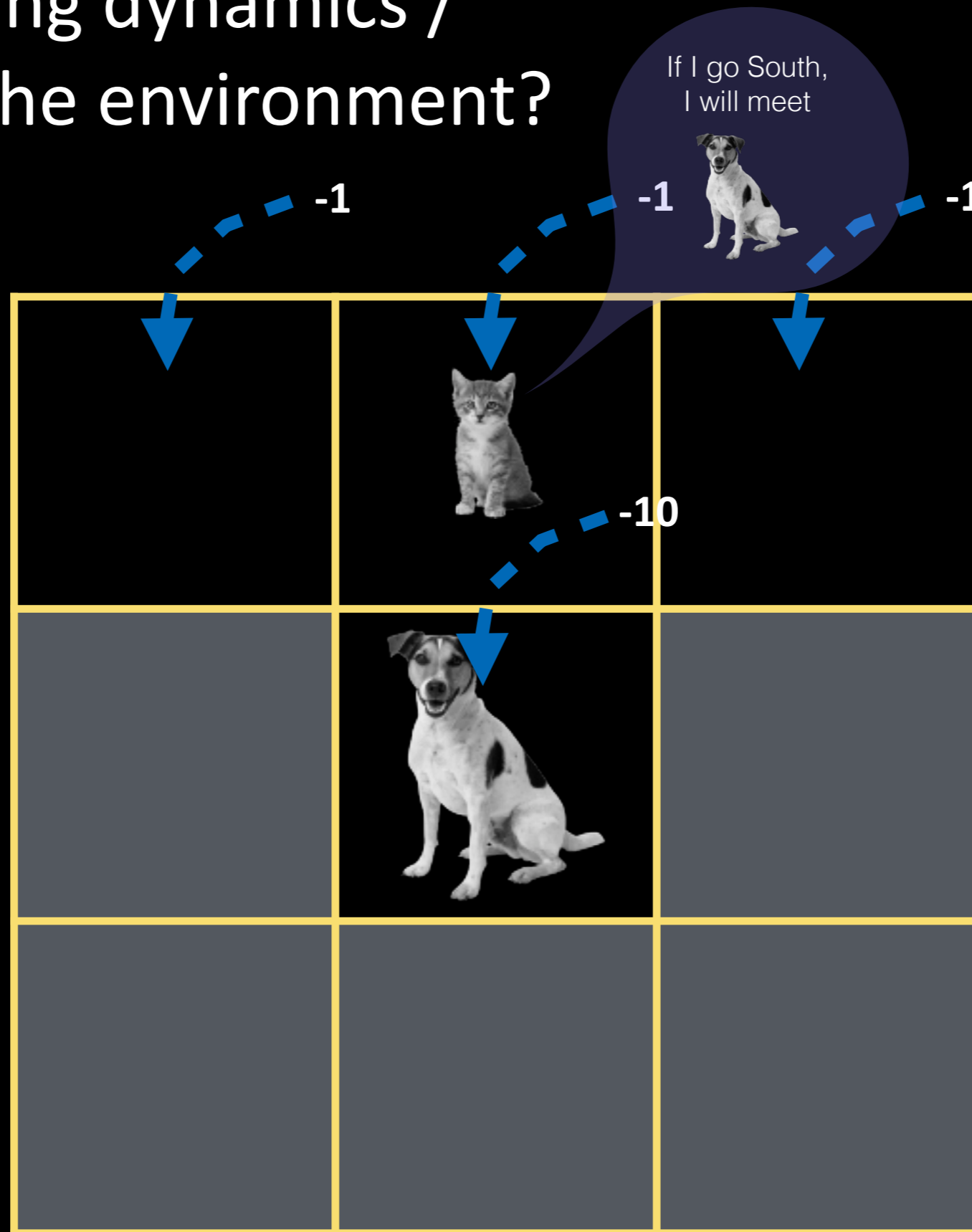
$E\{R\}$	$E\{R\}$	$E\{R\}$
$E\{R\}$	 $E\{R\}$	$E\{R\}$
$E\{R\}$ home	$E\{R\}$	$E\{R\}$

expected long term value of taking
 some action in each state, then behaving
 using some action selection scheme?



E{R}	E{R}	E{R}
E{R} E{R}	E{R} E{R}	E{R} E{R}
E{R}	E{R}	E{R}
E{R}	E{R}	E{R}
E{R} E{R}	E{R} E{R}	E{R} E{R}
E{R}	E{R}	E{R}
E{R} E{R}	E{R} E{R}	E{R} E{R}
E{R}	E{R}	E{R}
E{R} h E{R}	E{R} E{R}	E{R} E{R}
E{R}	E{R}	E{R}

modelling dynamics / mapping the environment?



prediction problem

learn to predict expected long
term reward/value

control problem

learn to find the optimal action
selection scheme/policy



policy: action selection

value: how good is an action/state










model: predict next state/reward
to look ahead/plan

types of RL agents?

value based

$E\{R\}$	$E\{R\}$	$E\{R\}$
$E\{R\}$ $E\{R\}$	$E\{R\}$ $E\{R\}$	$E\{R\}$ $E\{R\}$
$E\{R\}$	$E\{R\}$	$E\{R\}$
$E\{R\}$	 $E\{R\}$	$E\{R\}$
$E\{R\}$ $E\{R\}$	$E\{R\}$ $E\{R\}$	$E\{R\}$ $E\{R\}$
$E\{R\}$	$E\{R\}$	$E\{R\}$
$E\{R\}$ h $E\{R\}$	$E\{R\}$ $E\{R\}$	$E\{R\}$ $E\{R\}$
$E\{R\}$	$E\{R\}$	$E\{R\}$

policy based

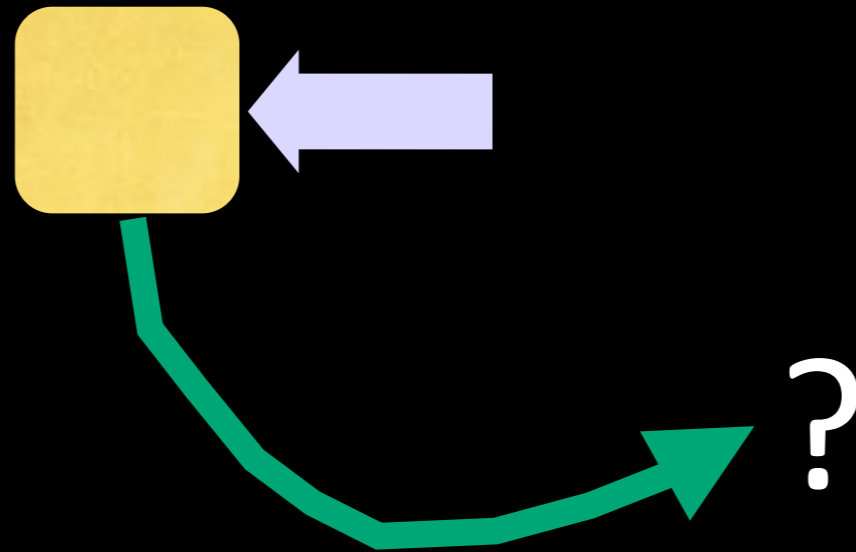
model based

value/policy
+
model of dynamics

both value and policy

**we will focus on value
based RL in the first half**

action selection?



expected return for carrying out an
action is its **value**

values of each possible action in the
current state helps select actions!

**policy can be
derived from value
(e.g. act greedily)**

but what are
these values?

<<expected returns **unknown**>>

<<actions based on **unknowns**>>

value can be **estimated** by
sampling environment
while acting using some policy

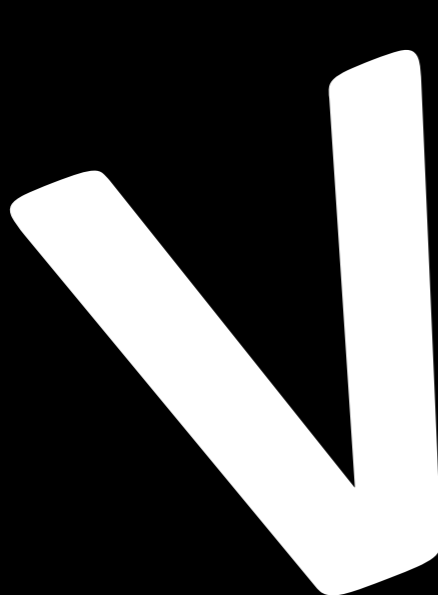
e.g. act, accumulate new
reward (ground truth), and update

Q

	a	b	c
1	2	0	1
2	3	0	-1
3	-5	6	2
4	2	3	1
.	.	.	.
.	.	.	.
.	.	.	.
n	7	8	7

agent maintains **values**
for **actions within each state**

selects actions using these values under some
“policy”

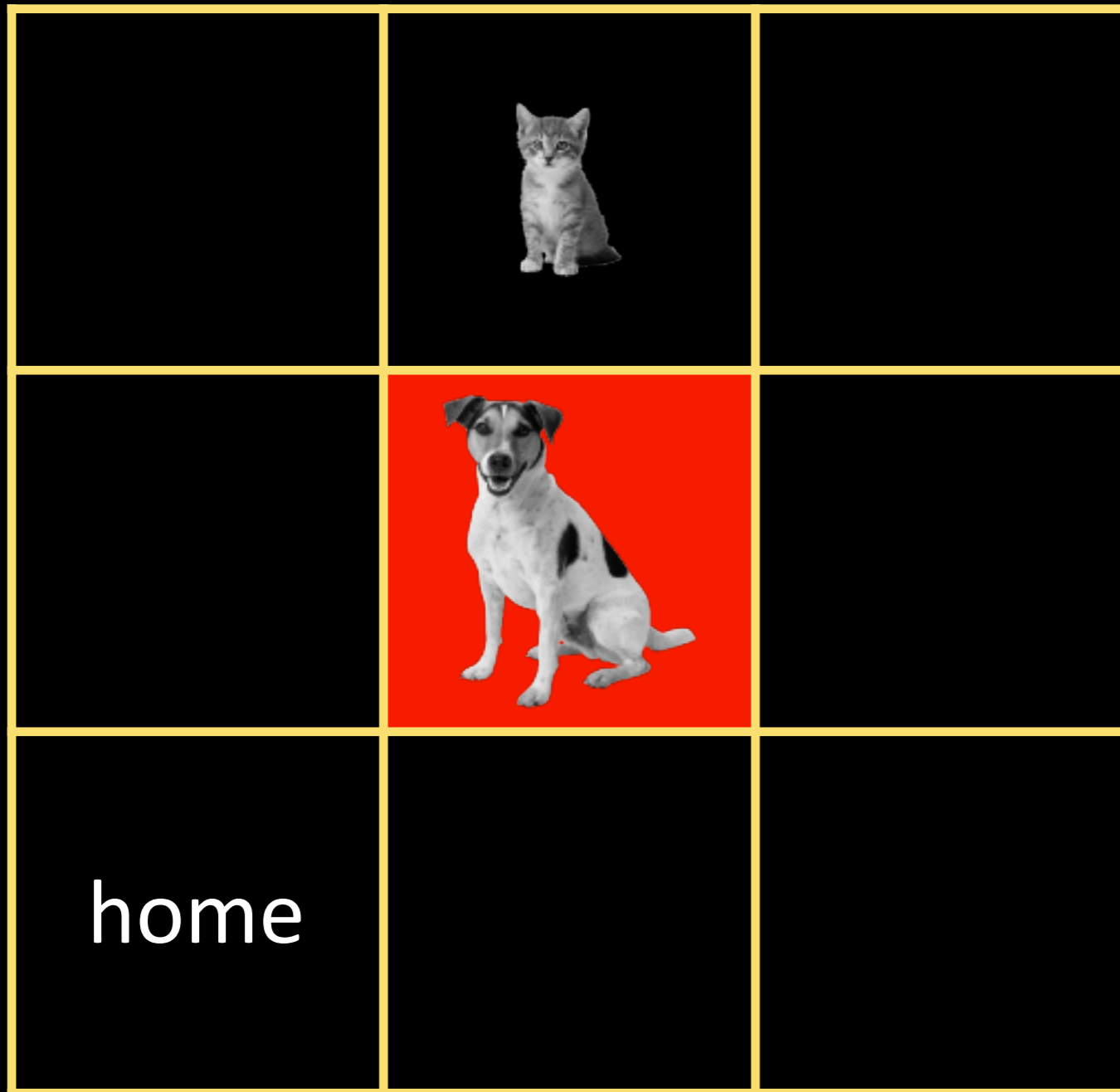


1	2
2	3
3	-5
4	2
.	.
.	.
.	.
n	7

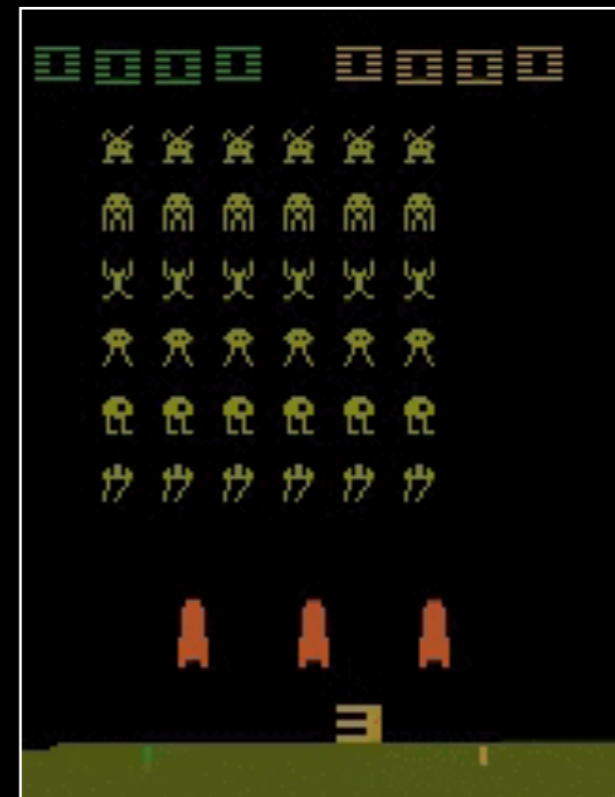
agent maintains state values

selects actions using these values under some
“policy”

but... agent needs a model of the environment!

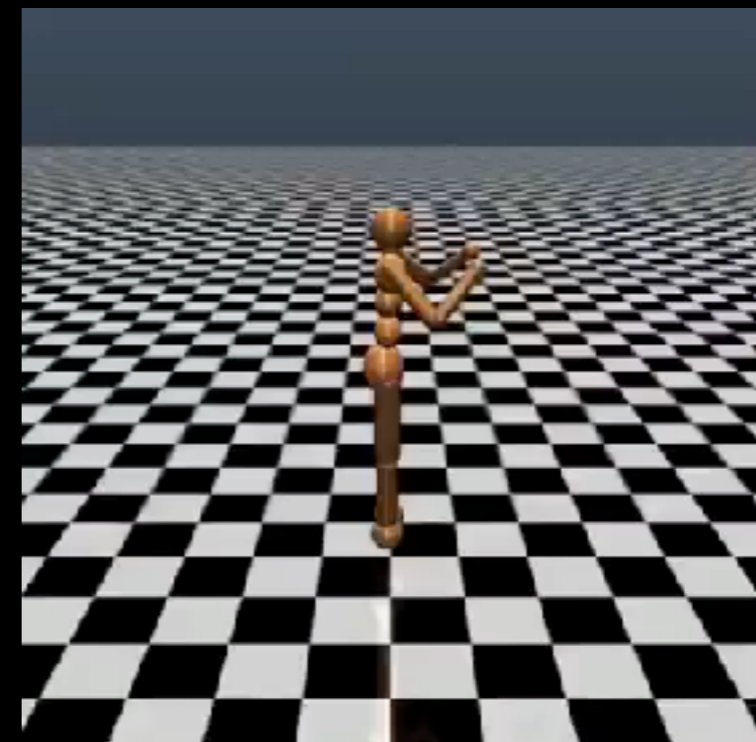


9 states



10^{16992} (pixels)

10^{308} (ram)

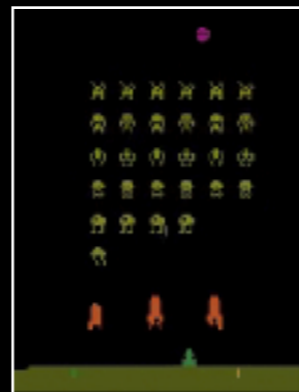
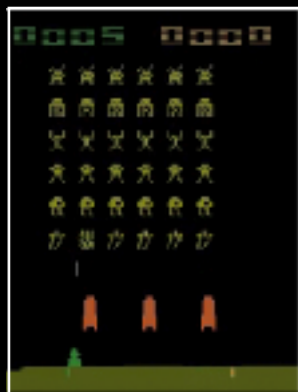


continuous!

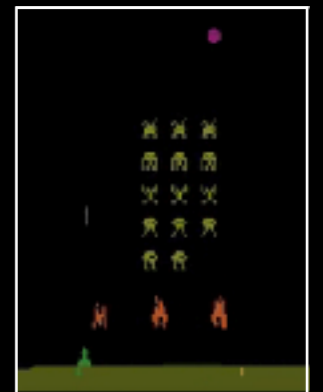
extract features
that help
generalise across states



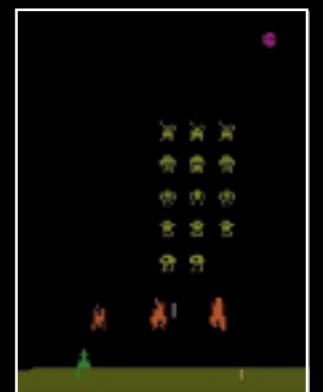
⋮



⋮



⋮



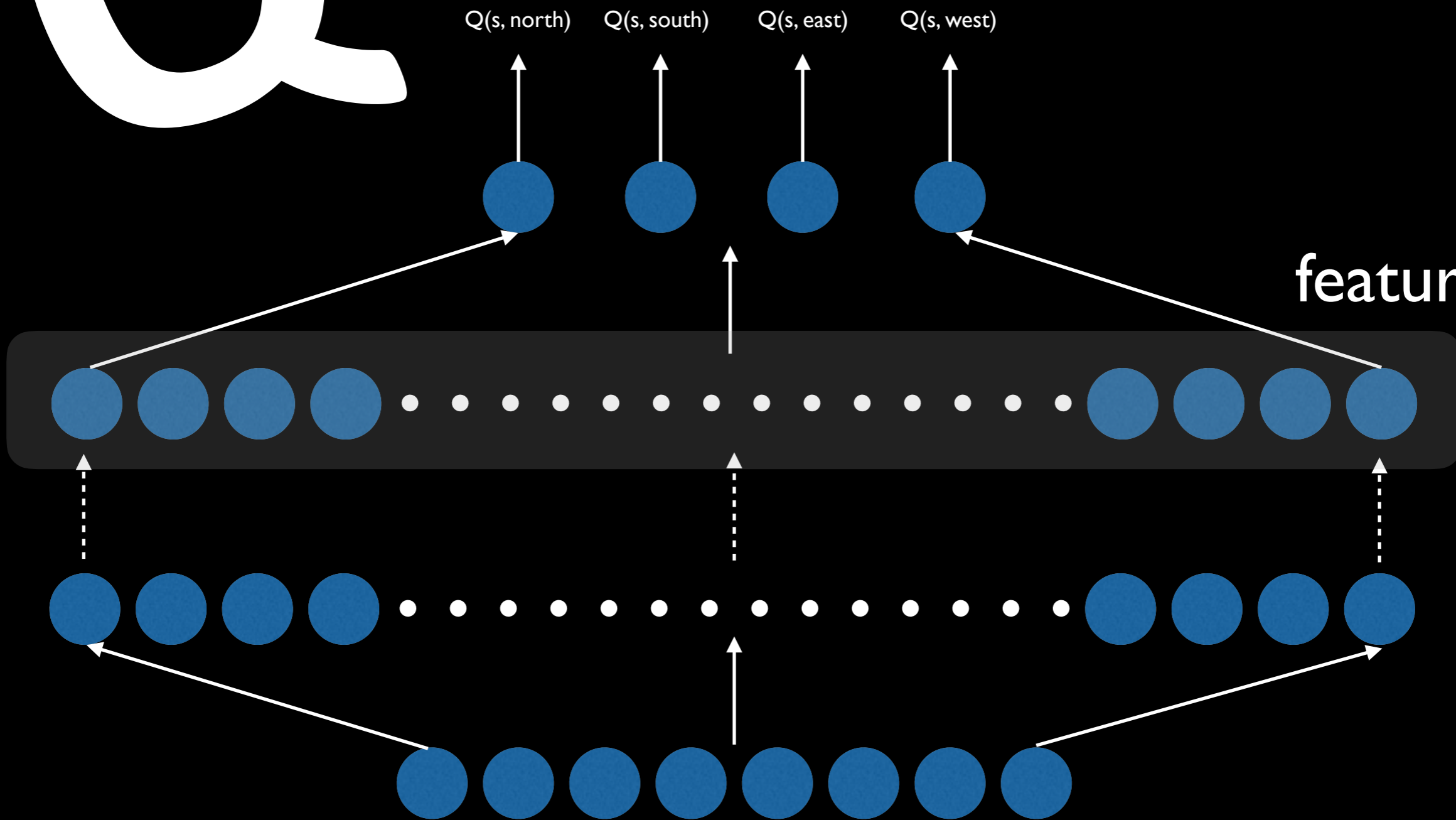
Q

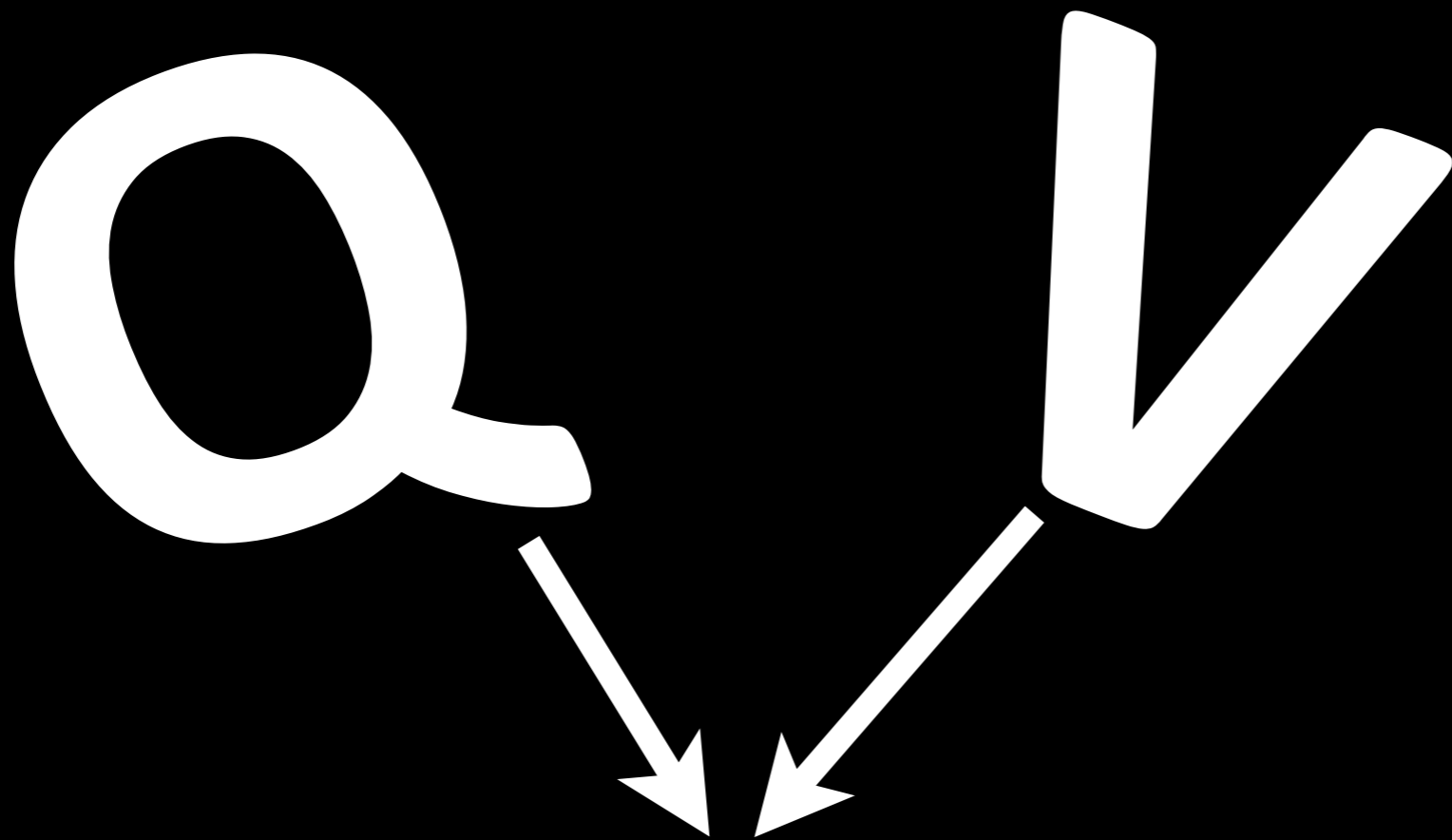
action values given state

$Q(s, \text{north})$ $Q(s, \text{south})$ $Q(s, \text{east})$ $Q(s, \text{west})$

features

state s





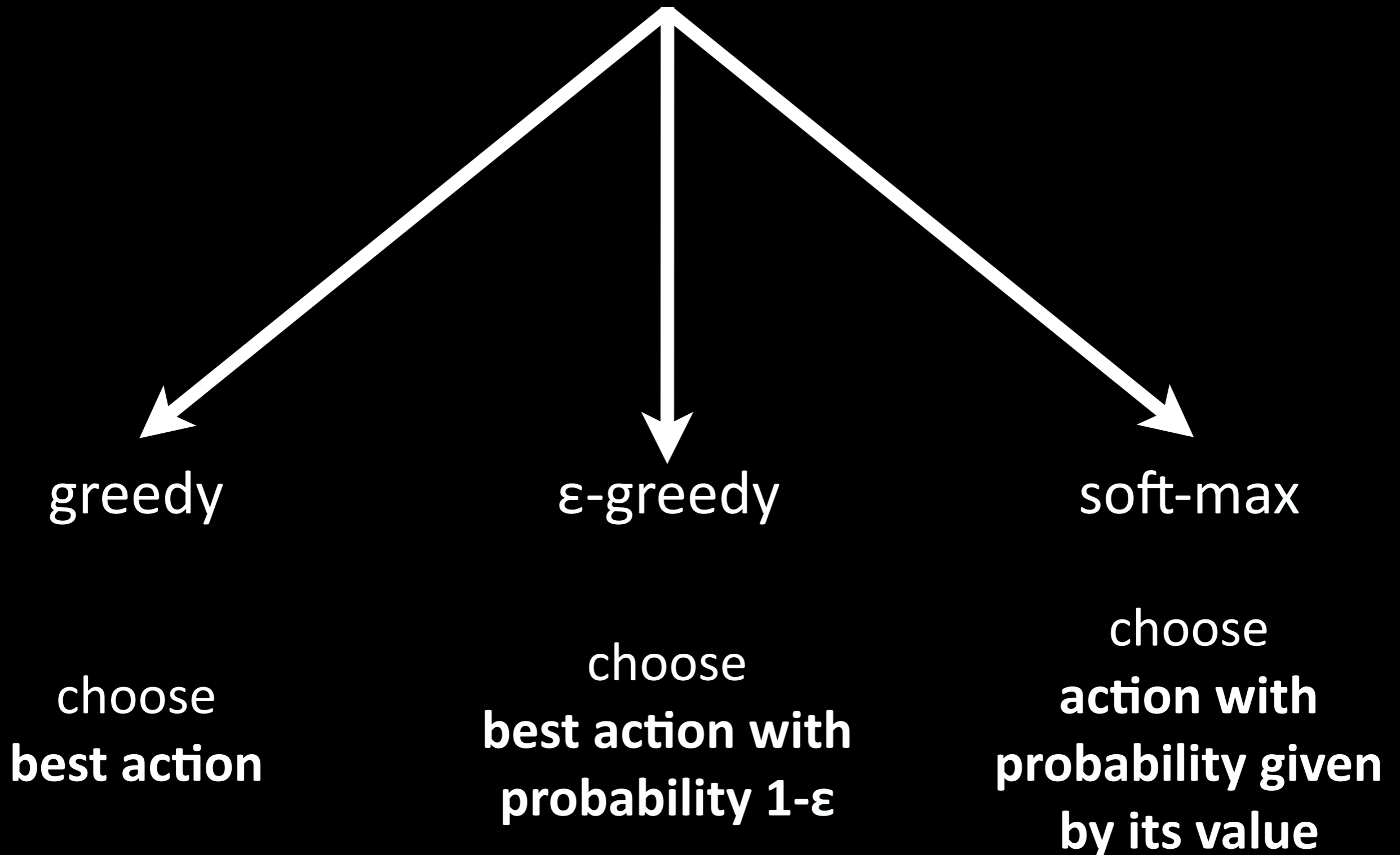
$E\{R_t\}$

policy?

probability of choosing
an action in state/feature
representation thereof

π $Q^\pi(s, a)$
 $V^\pi(s)$

usual policies



exploration vs. exploitation

trial and error

game play: try new moves

ads: try new ads

a/b testing:

try new website feature

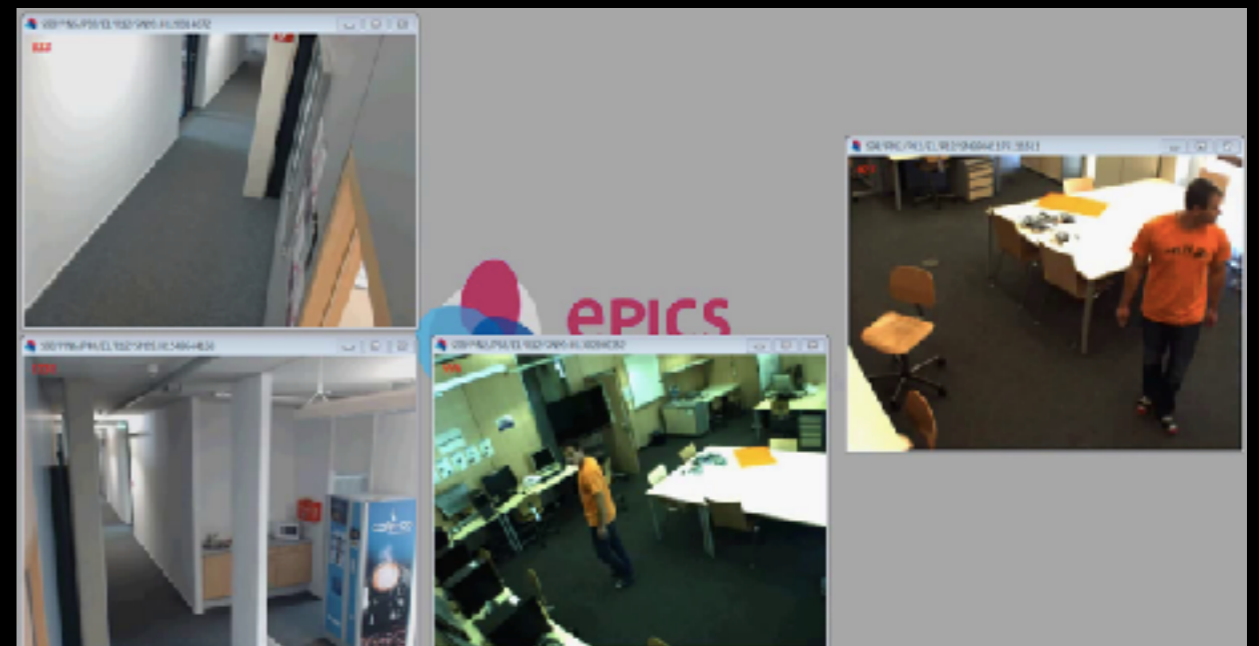
smart camera networks:

try new comm. protocol

Static, dynamic and adaptive heterogeneity in socio-economic distributed smart camera networks, P. R. Lewis, L. Esterle, A. Chandra, B. Rinner, J. Torresen, and X. Yao, ACM Transactions on Autonomous and Adaptive Systems (TAAS), ACM, 2015.



Yamaguchi先生, http://en.wikipedia.org/wiki/File:Las_Vegas_slot_machines.jpg



V*

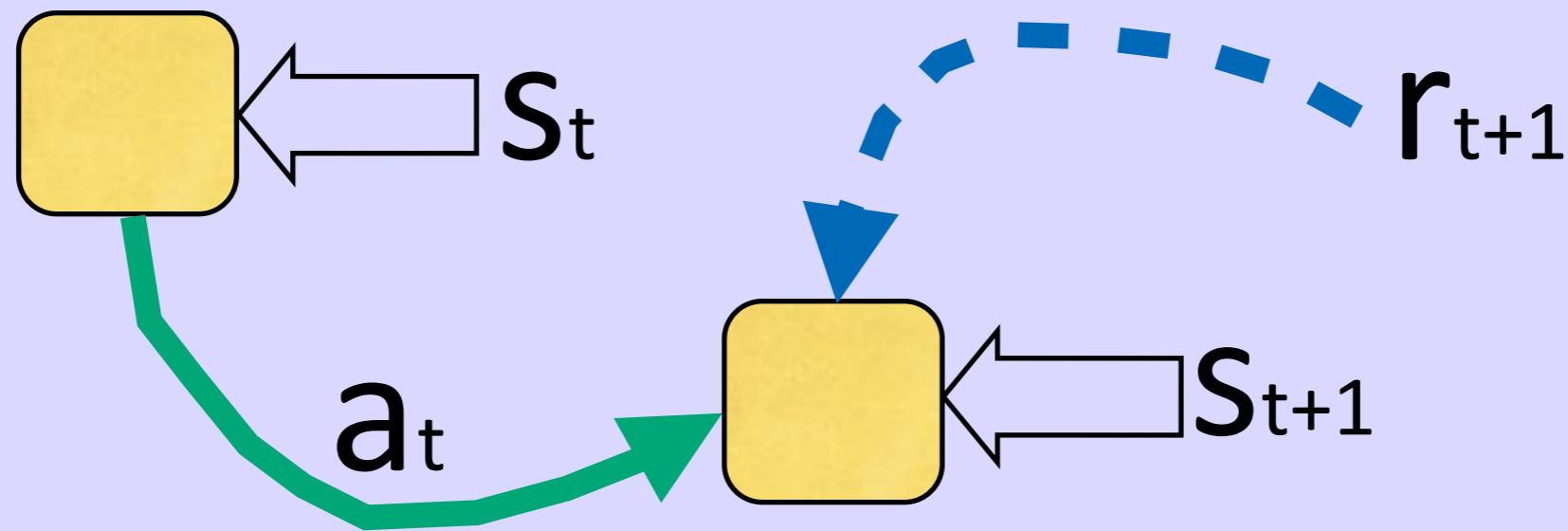
$$V^*(s) = \max_{\pi} V^{\pi}(s)$$

Q*

$$Q^*(s, a) = \max_{\pi} Q^{\pi}(s, a)$$

π^*

$$\pi^*(a|s) = \begin{cases} 1 & \text{if } a = \arg \max_a Q^*(s, a) \\ 0 & \text{otherwise} \end{cases}$$



estimation?

<<use currently visible returns to update values of where you are coming from>>

the current state (or state-action pair) has an **estimated value** (say zero/random initially), which can be used **together with r_{t+1}** to update **value** of previous state (or state-action pair)

i.e.

fraction of (currently visible returns - old value)

+

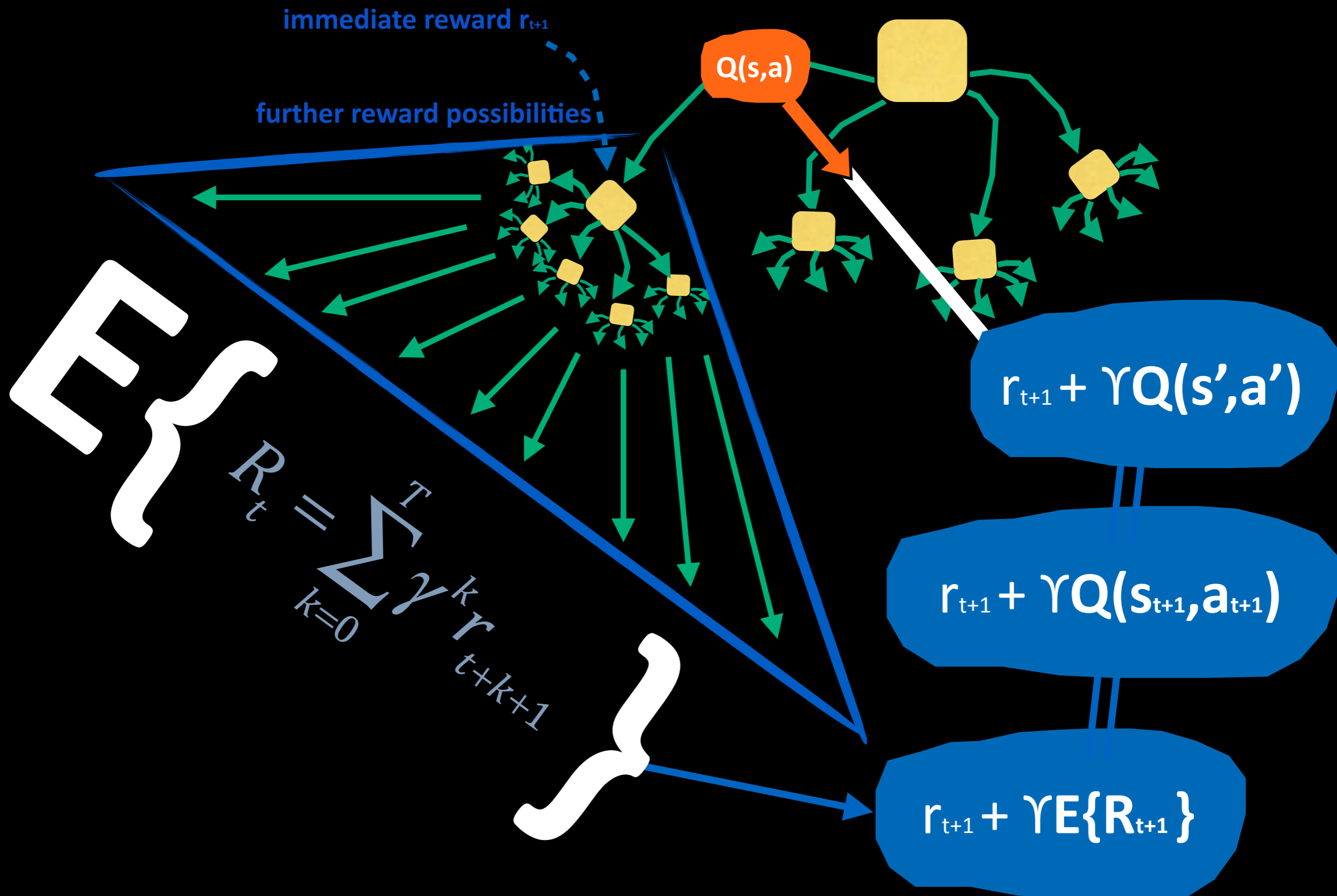
old value



new value

(1-fraction) old value + fraction curr. vis. returns





under some policy $\pi(a|s)$

$$V(s) \leftarrow V(s) + \alpha (r_s^a + \gamma V(s') - V(s))$$

e.g.

under some policy $\pi(a|s)$

$$Q(s, a) \leftarrow Q(s, a) + \alpha (r_s^a + \gamma Q(s', a') - Q(s, a))$$

e.g. update
a lookup table maintaining
expected returns

	a	b	c
1	2	0	1
2			-1
3	5		2
4	2	3	1
.	.	.	.
.	.	.	.
.	.	.	.
n	7	8	7

1	2
2	3
3	5
4	2
.	.
.	.
.	.
n	7

$$Q(s,a) \leftarrow Q(s,a) + \alpha(r_s^a + \gamma Q(s',a') - Q(s,a))$$

let's play with a version of the
above update rule:

$$Q(s,a) \leftarrow Q(s,a) + \alpha(r_s^a + \gamma \max_{a'} Q(s',a') - Q(s,a))$$

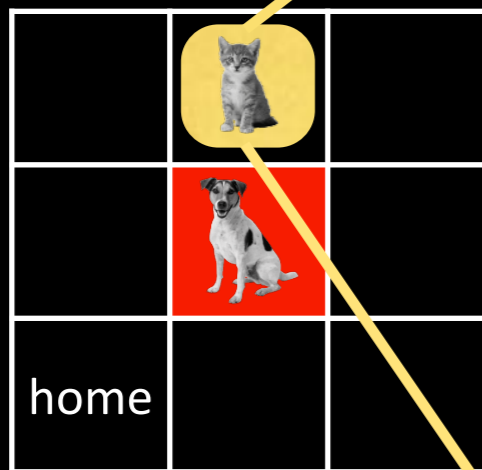
indicates a' to be the action
with maximum value in next
state s'

let's play with a version of the
above update rule:

$$Q(s, a) \leftarrow Q(s, a) + \alpha (r_s^a + \gamma \max_{a'} Q(s', a') - Q(s, a))$$

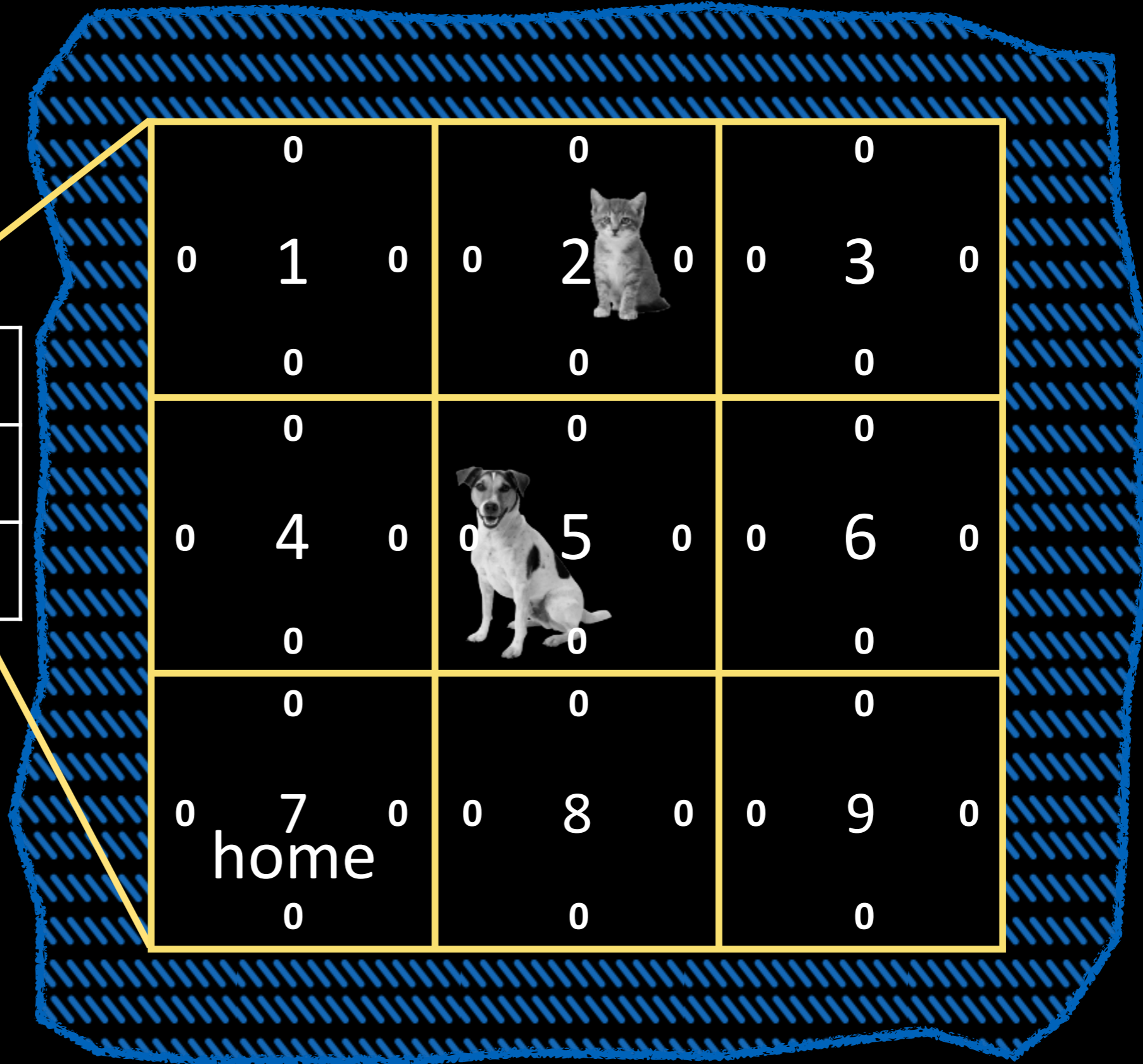

our toy problem

lookup table



	N	S	E	W
1	0	0	0	0
2	0	0	0	0
3	0	0	0	0
4	0	0	0	0
5	0	0	0	0
6	0	0	0	0
7	0	0	0	0
8	0	0	0	0
9	0	0	0	0

reward
structure?



move...

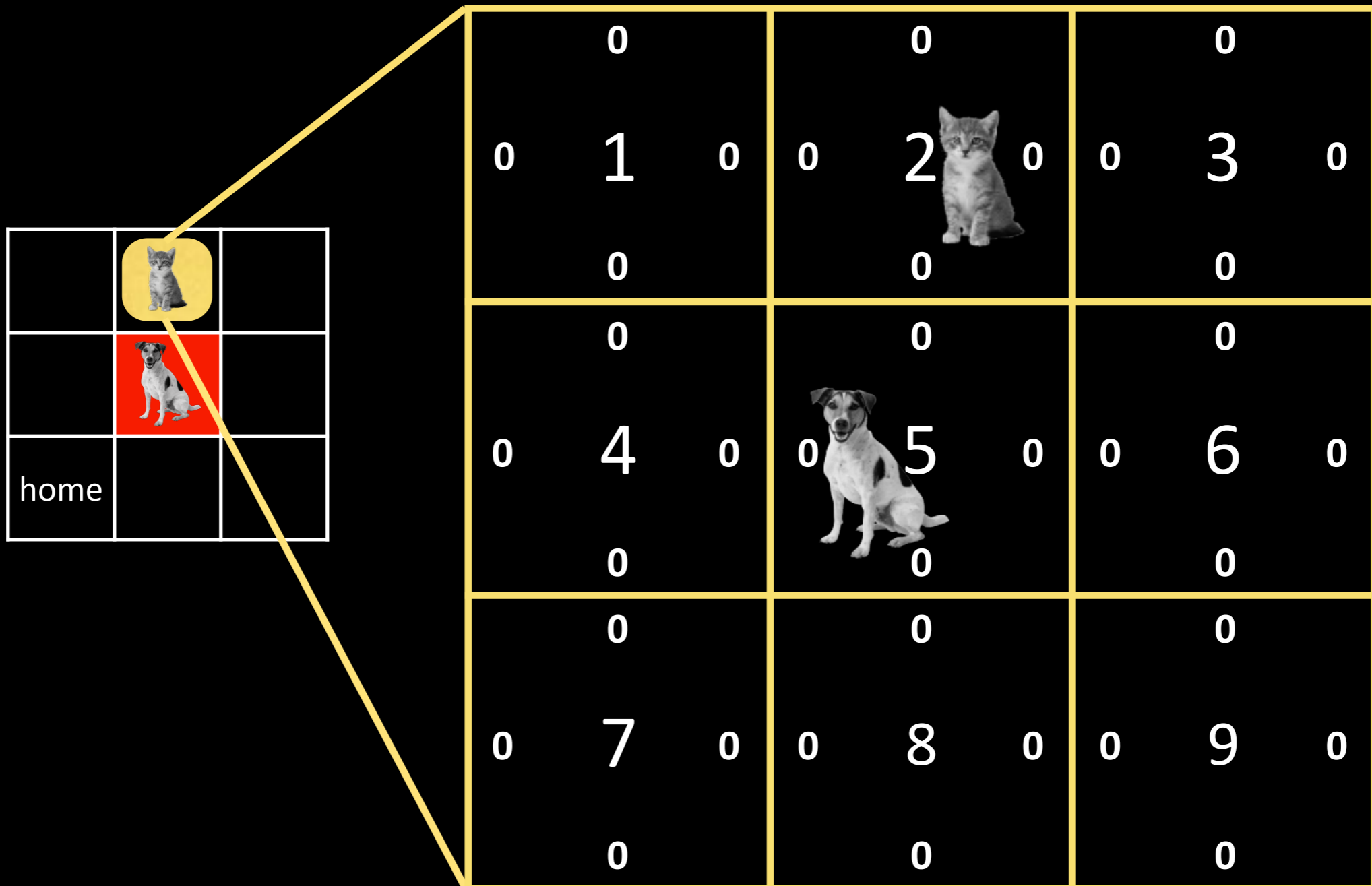
to any cell except 5 and 7:
-1

out of bounds:
-5



to 5:
-10

to 7/home:
10

let's fix $\alpha = 0.1$, $\gamma = 0.5$



$$Q(s,a) \leftarrow Q(s,a) + \alpha(r_s^a + \gamma \max_{a'} Q(s',a') - Q(s,a))$$

0	0	0
0 1 0	0 2 0 	0 3 0
0	0	0
0 4 0	0  5 0	0 6 0
0	0	0
0 7 0	0 8 0	0 9 0
home		
0	0	0

$$\alpha = 0.1$$

$$\gamma = 0.5$$

say ϵ -greedy policy...

episode 1 begins...



$$Q(s,a) \leftarrow Q(s,a) + \alpha(r_s^a + \gamma \max_{a'} Q(s',a') - Q(s,a))$$

0 0 1 0 0	0 ? 2 0 0	0 0 3 0 0
0 0 4 0 0	0 0 5 0 0	0 0 6 0 0
0 0 7 0 home 0	0 0 8 0 0	0 0 9 0 0

A blue dashed arrow labeled '-1' points from the top of the first column to the top of the second column. A solid blue arrow points from the top of the first column to the kitten image in the first cell.

$\alpha = 0.1$
 $\gamma = 0.5$


$$Q(s,a) \leftarrow Q(s,a) + \alpha(r_s^a + \gamma \max_{a'} Q(s',a') - Q(s,a))$$

0 0 1  0 0	-0.1 0 2 0 0	0 0 3 0 0
0 0 4 0 0	0  0 0 5 0 0	0 0 6 0 0
0 0 7 home 0	0 0 8 0 0	0 0 9 0 0

$$\alpha = 0.1$$

$$\gamma = 0.5$$

$$Q(s,a) \leftarrow Q(s,a) + \alpha(r_s^a + \gamma \max_{a'} Q(s',a') - Q(s,a))$$

0	0	0
0	1  0	-0.1 2 0
0	0	0
0	0	0
0	4 0	0 5 0
0	0	0
0	7 0	0 8 0
0	home	0 9 0
0	0	0

$$\alpha = 0.1$$

$$\gamma = 0.5$$



$$Q(s,a) \leftarrow Q(s,a) + \alpha(r_s^a + \gamma \max_{a'} Q(s',a') - Q(s,a))$$

?	0	0
0 1 0 	-0.1 2 0	0 3 0
0	0	0
0 4 0	0 5 0 	0 6 0
0	0	0
0 7 0 home	0 8 0	0 9 0
0	0	0

$\alpha = 0.1$
 $\gamma = 0.5$



-5

$$Q(s,a) \leftarrow Q(s,a) + \alpha(r_s^a + \gamma \max_{a'} Q(s',a') - Q(s,a))$$

-0.5 0 1  0 0	0 -0.1 2 0 0	0 0 3 0 0
0 0 4 0 0	0 0  5 0 0	0 0 6 0 0
0 0 7 home 0	0 0 8 0 0	0 0 9 0 0

$\alpha = 0.1$
 $\gamma = 0.5$

$$Q(s,a) \leftarrow Q(s,a) + \alpha(r_s^a + \gamma \max_{a'} Q(s',a') - Q(s,a))$$

-0.5 0 1 0  0	0 -0.1 2 0 0	0 0 3 0 0
0 0 4 0 0	0 0 5 0  0	0 0 6 0 0
0 0 7 0 home 0	0 0 8 0 0	0 0 9 0 0



$\alpha = 0.1$
 $\gamma = 0.5$

$$Q(s,a) \leftarrow Q(s,a) + \alpha(r_s^a + \gamma \max_{a'} Q(s',a') - Q(s,a))$$

-0.5	0	0	0	0	0	0	0	0
0	1	0	-0.1	2	0	0	3	0
?	?	0	-1	0	0	0	0	0
0	4	0	0	5	0	0	6	0
0	0	0	0	0	0	0	0	0
0	7	0	0	8	0	0	9	0
0	home	0	0	0	0	0	0	0

$\alpha = 0.1$
 $\gamma = 0.5$



$$Q(s,a) \leftarrow Q(s,a) + \alpha(r_s^a + \gamma \max_{a'} Q(s',a') - Q(s,a))$$

-0.5	0	0
0 1 0	-0.1 2 0	0 3 0
-0.1	0	0
0	0	0
0 4  0	0  5 0	0 6 0
0	0	0
0 7 0	0 8 0	0 9 0
home	0	0
0	0	0

$$\alpha = 0.1$$

$$\gamma = 0.5$$

$$Q(s,a) \leftarrow Q(s,a) + \alpha(r_s^a + \gamma \max_{a'} Q(s',a') - Q(s,a))$$

-0.5	0	0
0 1 0	-0.1 2 0	0 3 0
-0.1	0	0
0	0	0
0 4  0	0  5 0	0 6 0
0	0	0
0 7 0	0 8 0	0 9 0
home	0	0
0	0	0

$$\alpha = 0.1$$



$$\gamma = 0.5$$

$$Q(s,a) \leftarrow Q(s,a) + \alpha(r_s^a + \gamma \max_{a'} Q(s',a') - Q(s,a))$$

	-0.5		0		0
0	1	0	-0.1	2	0
	-0.1		0		-10
0	4	?	0	5	0
	0		0		0
0	7	0	0	8	0
home			0		0
	0		0		0

$\alpha = 0.1$
 $\gamma = 0.5$

$$Q(s,a) \leftarrow Q(s,a) + \alpha(r_s^a + \gamma \max_{a'} Q(s',a') - Q(s,a))$$

-0.5	0	0
0 1 0	-0.1 2 0	0 3 0
-0.1	0	0
0	0	0
0 4 -1	0  5  0	0 6 0
0	0	0
0 7 0	0 8 0	0 9 0
home	0	0
0	0	0

$$\alpha = 0.1$$

$$\gamma = 0.5$$

$$Q(s,a) \leftarrow Q(s,a) + \alpha(r_s^a + \gamma \max_{a'} Q(s',a') - Q(s,a))$$

-0.5	0	0
0 1 0	-0.1 2 0	0 3 0
-0.1	0	0
0	0	0
0 4 -1	0 5 0	0 6 0
0	0	0
0	0	0
0 7 0	0 8 0	0 9 0
home	0	0
0	0	0

$$\alpha = 0.1$$

$$\gamma = 0.5$$


$$Q(s,a) \leftarrow Q(s,a) + \alpha(r_s^a + \gamma \max_{a'} Q(s',a') - Q(s,a))$$

-0.5	0	0
0 1 0	-0.1 2 0	0 3 0
-0.1	0	0
0	0	0
0 4 -1	0 5 0	0 6 0
0	?	-1
0	0	0
0 7 0	0 8 0	0 9 0
home	0	0
0	0	0

$$\alpha = 0.1$$

$$\gamma = 0.5$$


$$Q(s,a) \leftarrow Q(s,a) + \alpha(r_s^a + \gamma \max_{a'} Q(s',a') - Q(s,a))$$

-0.5	0	0
0 1 0	-0.1 2 0	0 3 0
-0.1	0	0
0	0	0
0 4 -1	0  5 0	0 6 0
0	-0.1	0
0	0	0
0 7 0	0  8 0	0 9 0
home	0	0
0	0	0

$$\alpha = 0.1$$

$$\gamma = 0.5$$

$$Q(s,a) \leftarrow Q(s,a) + \alpha(r_s^a + \gamma \max_{a'} Q(s',a') - Q(s,a))$$

	-0.5		0		0
0	1	0	-0.1	2	0
	-0.1		0		0
0	4	-1	0	5	0
	0		 -0.1		0
0	7	0	0	8	0
home					9
	0		0		0

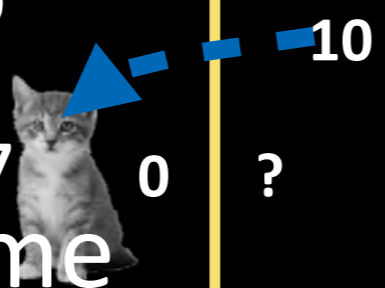
$$\alpha = 0.1$$

$$\gamma = 0.5$$

$$Q(s,a) \leftarrow Q(s,a) + \alpha(r_s^a + \gamma \max_{a'} Q(s',a') - Q(s,a))$$

	-0.5		0		0
0	1	0	-0.1	2	0
	-0.1		0		0
0	4	-1	0	5	0
	0			-0.1	
0	7	0	?	8	0
	home				
	0		0		0

$\alpha = 0.1$
 $\gamma = 0.5$



$$Q(s,a) \leftarrow Q(s,a) + \alpha(r_s^a + \gamma \max_{a'} Q(s',a') - Q(s,a))$$

-0.5	0	0
0 1 0	-0.1 2 0	0 3 0
-0.1	0	0
0	0	0
0 4 -1	0 5 0	0 6 0
0	 -0.1	0
0	0	0
0 7 0	1 8 0	0 9 0
home	0	0
0	0	0

$$\alpha = 0.1$$

$$\gamma = 0.5$$

episode 1 ends.

let's work out the next episode, starting at state 4

-0.5	0	0
0 1 0	-0.1 2 0	0 3 0
-0.1	0	0
0	0	0
0 4  -1	0  5 0	0 6 0
0	-0.1	0
0	0	0
0 7 0	1 8 0	0 9 0
home	0	0
0	0	0

$$\alpha = 0.1$$

$$\gamma = 0.5$$

go WEST and then SOUTH

how does the table change?

	-0.5		0		0		0	
0	1	0	-0.1	2	0	0	3	0
	-0.1		0		0		0	
	0		0		0		0	
-0.5	4	-1	0	5	0	0	6	0
	1		-0.1		0		0	
	0		0		0		0	
0	7	0	1	8	0	0	9	0
	0		0		0		0	

$$\alpha = 0.1$$

$$\gamma = 0.5$$

and the next episode,
starting at state 3

go WEST -> SOUTH -> WEST -> SOUTH

	-0.5		0		0		0	
0	1	0	-0.1	2	0	-0.1	3	0
	-0.1			-1			0	
	0			0			0	
-0.5	4	-1	-0.05	5	0	0	6	0
	1.9			-0.1			0	
	0			0			0	
0	7	0	1	8	0	0	9	0
	0			0			0	

$$\alpha = 0.1$$

$$\gamma = 0.5$$

over time, values will converge to optimal!

what we just saw was
some episodes of
Q-learning

values update towards value of **optimal** policy:
target comes from value of
assumed next best action

off-policy learning

SARSA-learning?

values update towards value of **current** policy:
target comes from value of
the actual next action

on-policy learning



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data not generated by target policy

data generated by target policy



$\epsilon: 0.1$
 $\gamma: 1.0$

Q

SARSA

Problem Decomposition

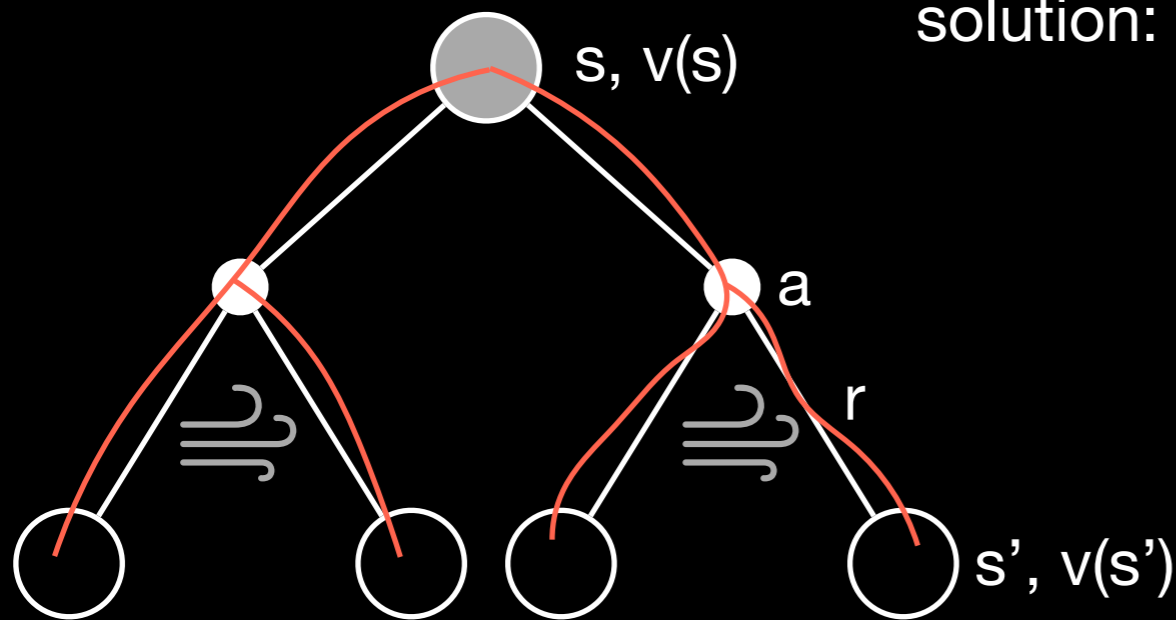
**nested
sub-problems**

**solution to sub-problem
informs
solution to whole problem**

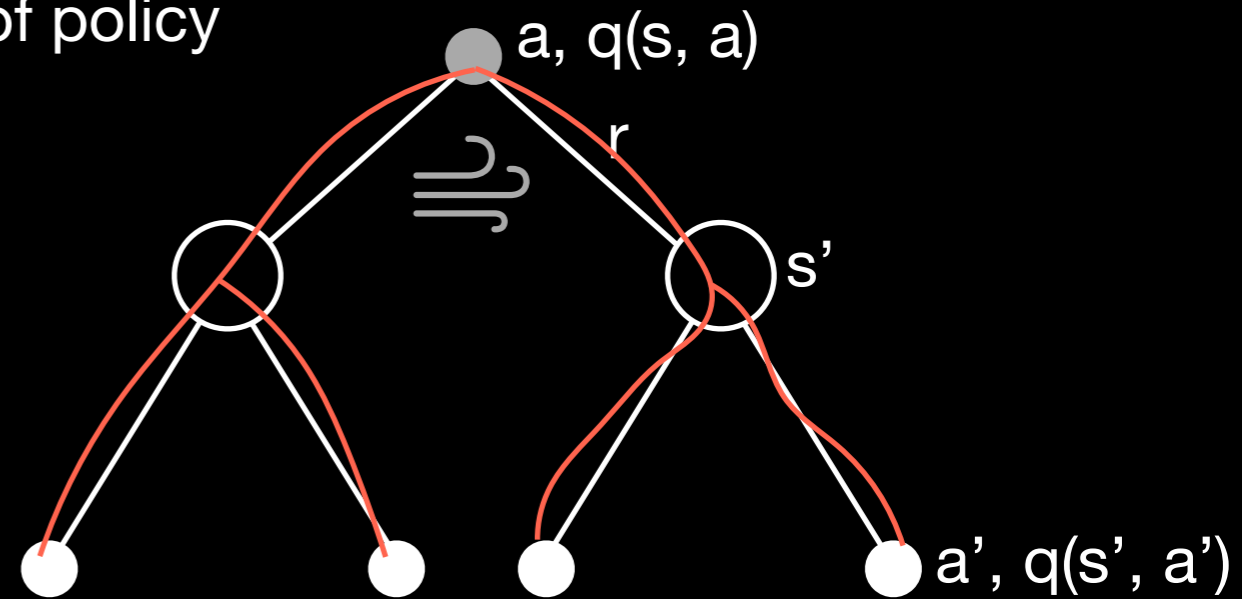


Bellman Expectation Backup

system of linear equations
solution: value of policy



Value of \bullet = $P(\text{path}) * \text{Value}(\text{path})$



Value of \bullet = $P(\text{path}) * \text{Value}(\text{path})$

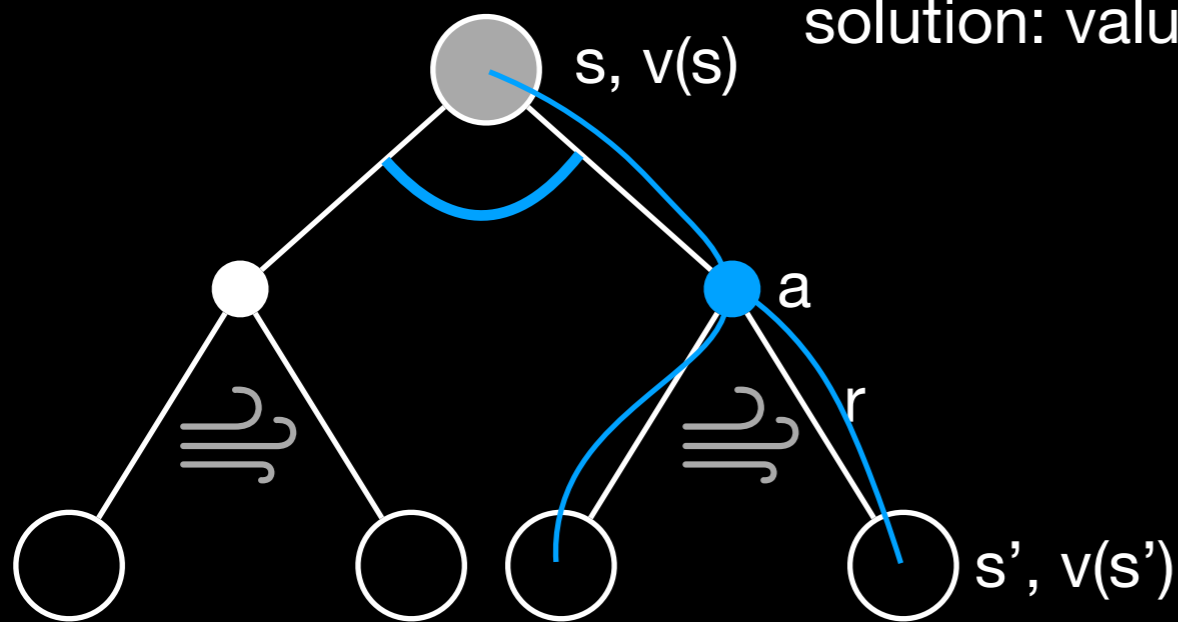
$$v_{\pi}(s) = \sum_a \pi(a|s) \left(r_s^a + \gamma \sum_{s'} P_{ss'}^a v_{\pi}(s') \right)$$

$$q_{\pi}(s, a) = r_s^a + \gamma \sum_{s'} P_{ss'}^a \sum_{a'} \pi(a'|s') q_{\pi}(s', a')$$

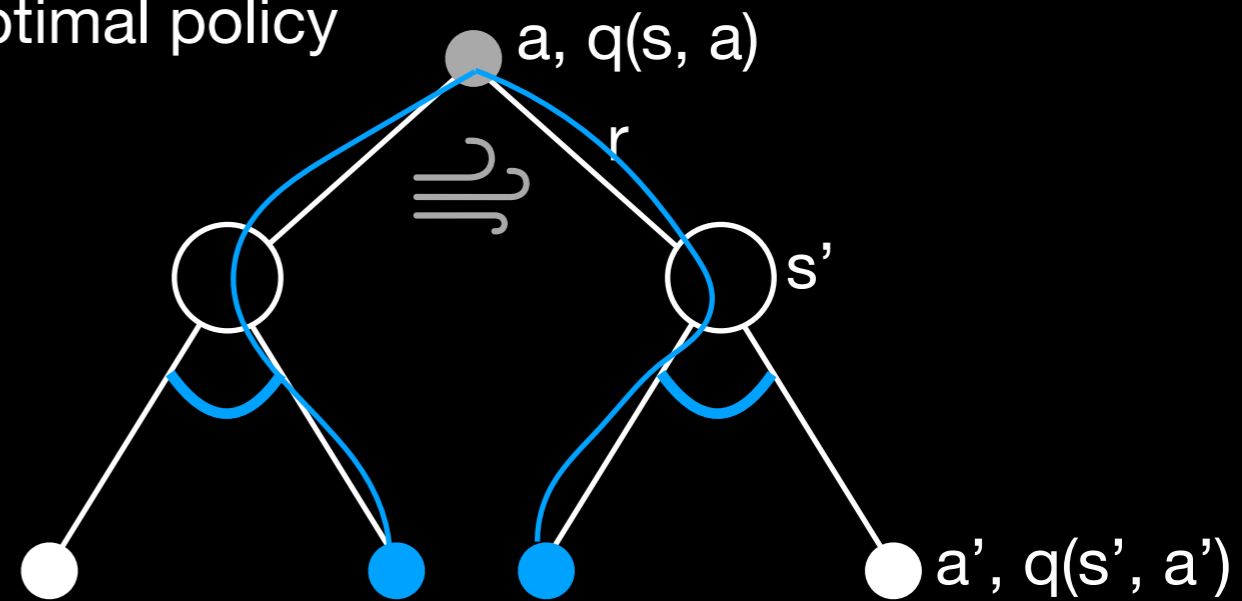
Bellman expectation equations
under a **given policy**

Bellman Optimality Backup

system of non-linear equations
solution: value of optimal policy



Value of  = $P(\text{path}) * \text{Value}(\text{path})$



Value of  = $P(\text{path}) * \text{Value}(\text{path})$

$$v_*(s) = \max_a \left(r_s^a + \gamma \sum_{s'} P_{ss'}^a v_*(s') \right)$$

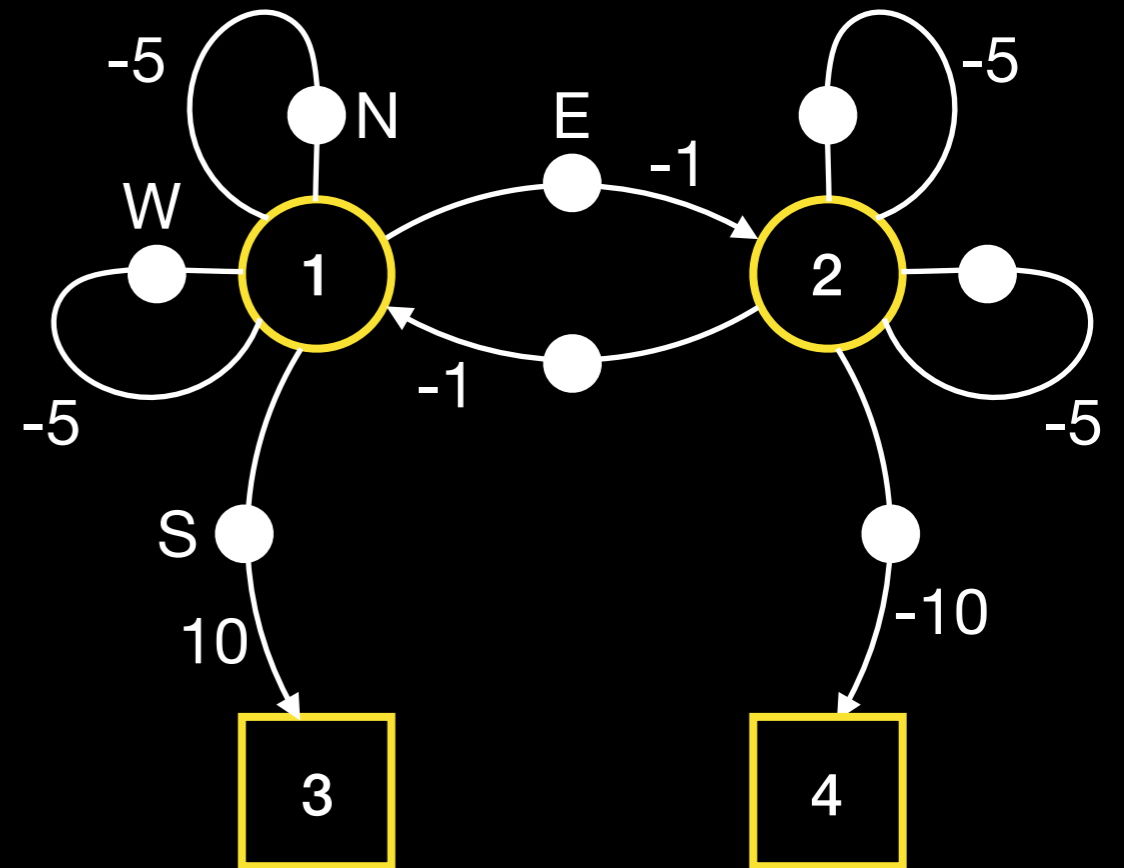
$$q_*(s, a) = r_s^a + \gamma \sum_{s'} P_{ss'}^a \max_{a'} q_*(s', a')$$

Bellman optimality equations
under **optimal policy**

Value Based

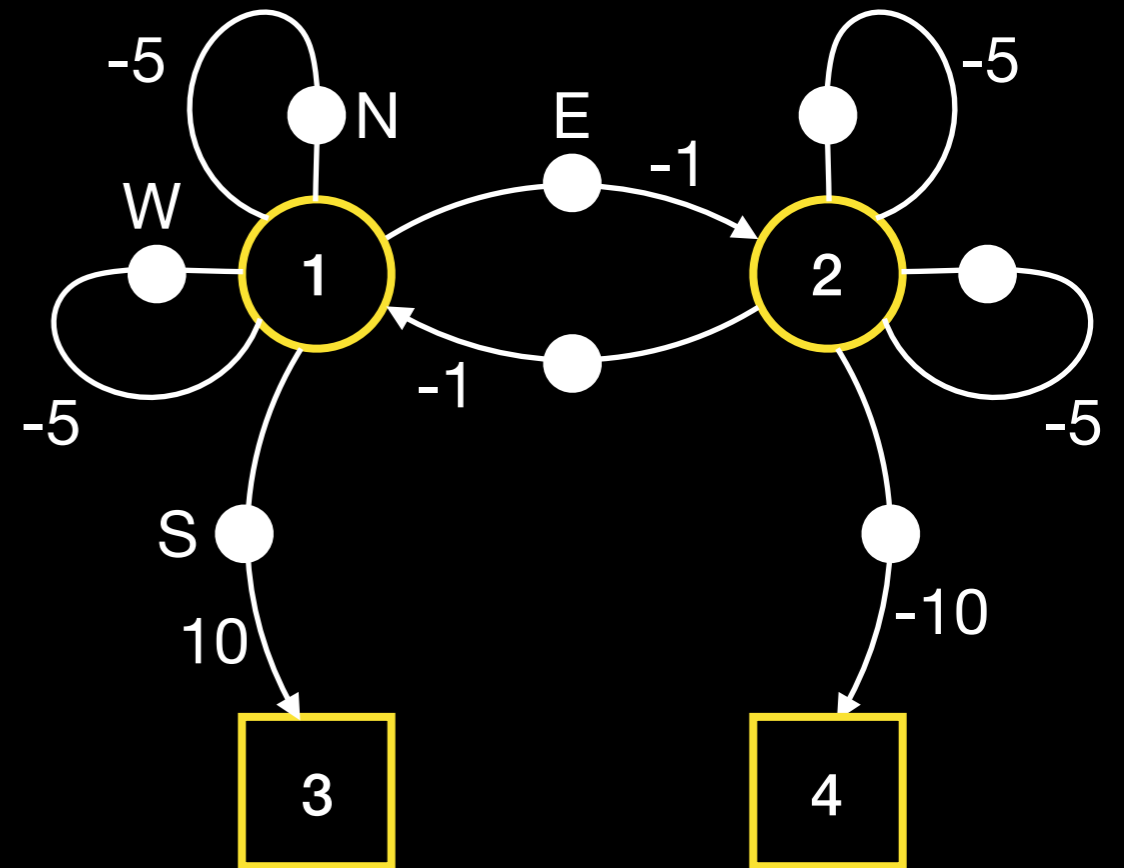
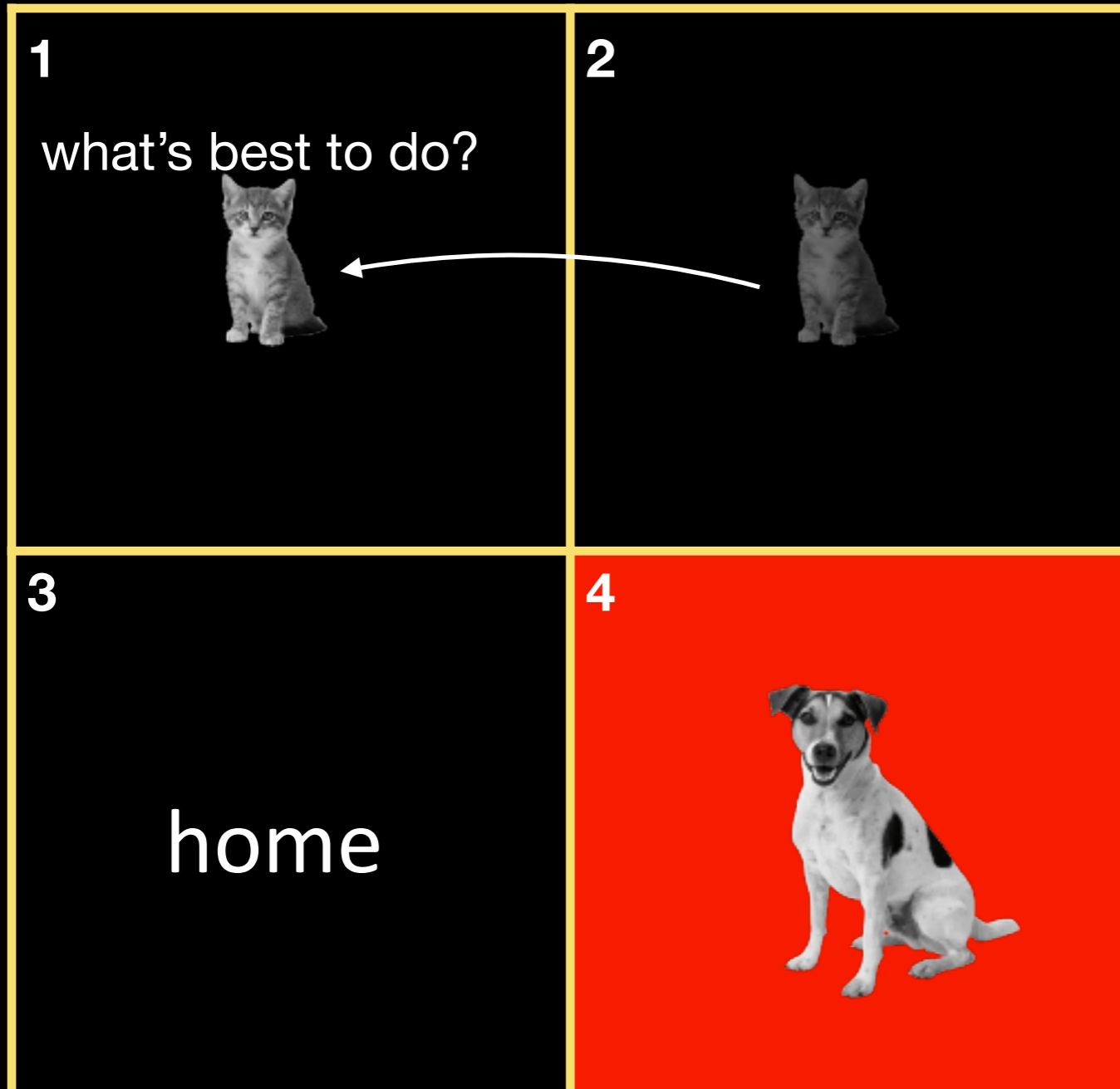
Dynamic Programming

...using Bellman equations as iterative updates

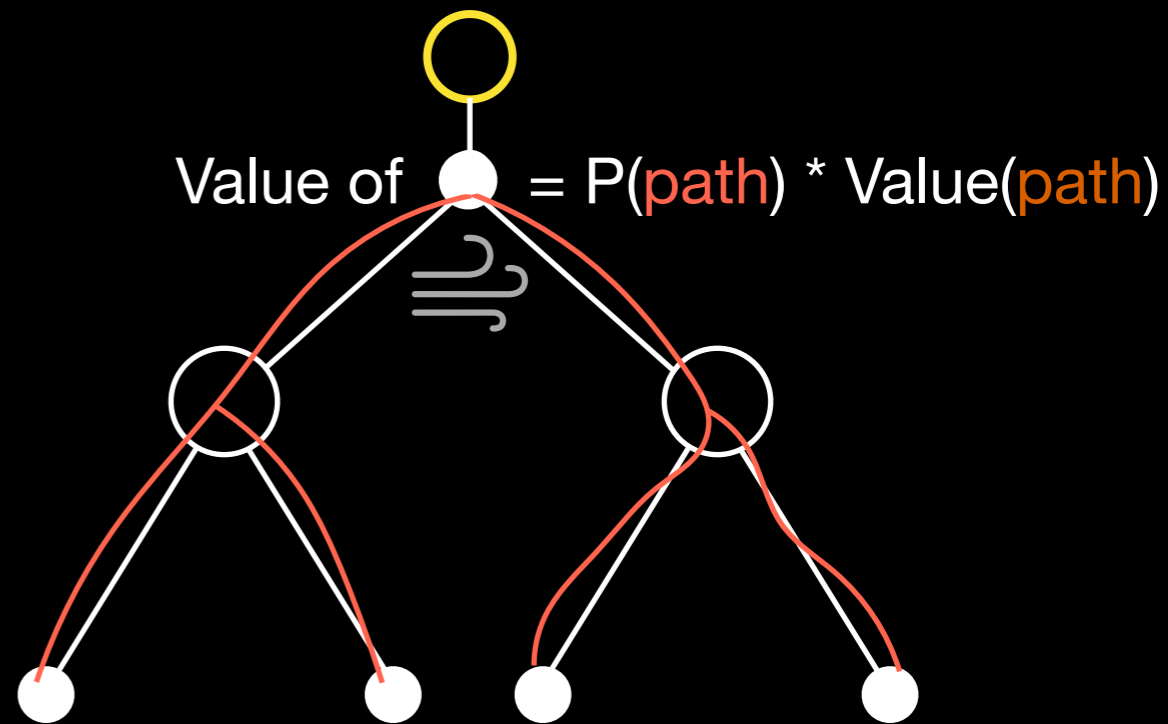
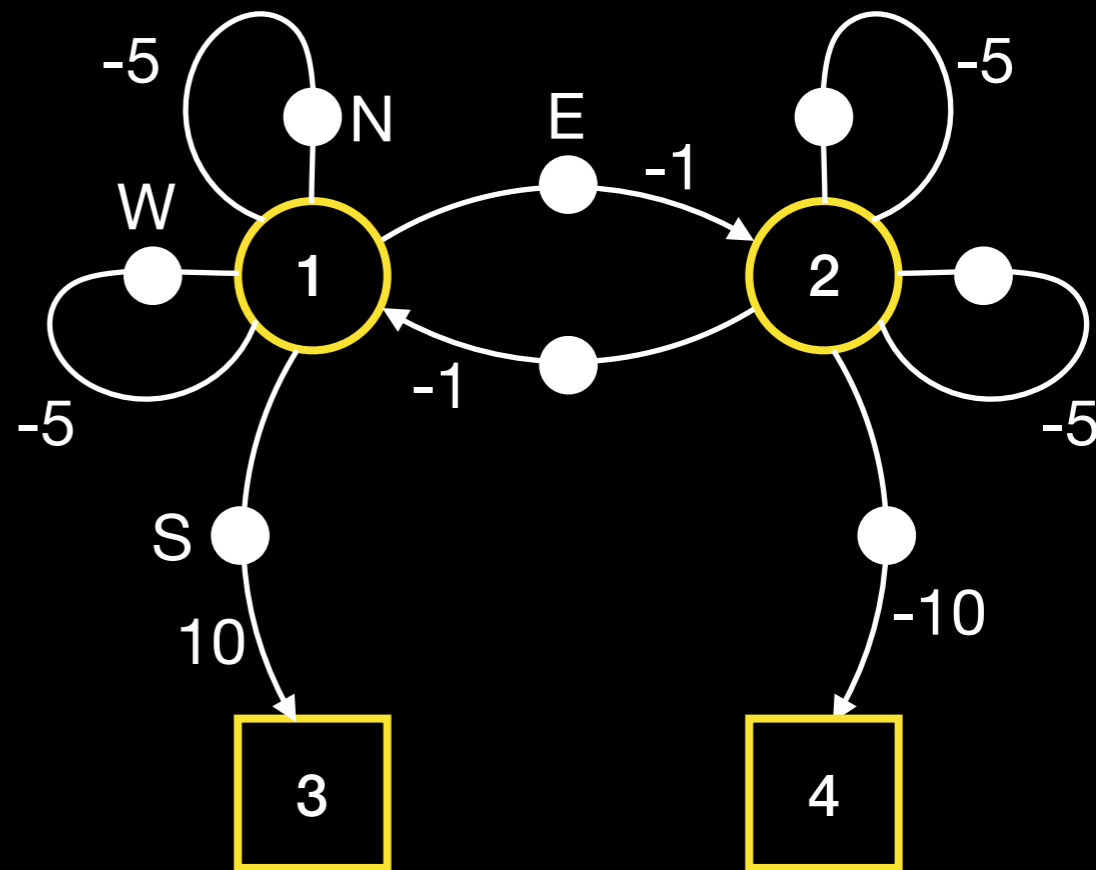
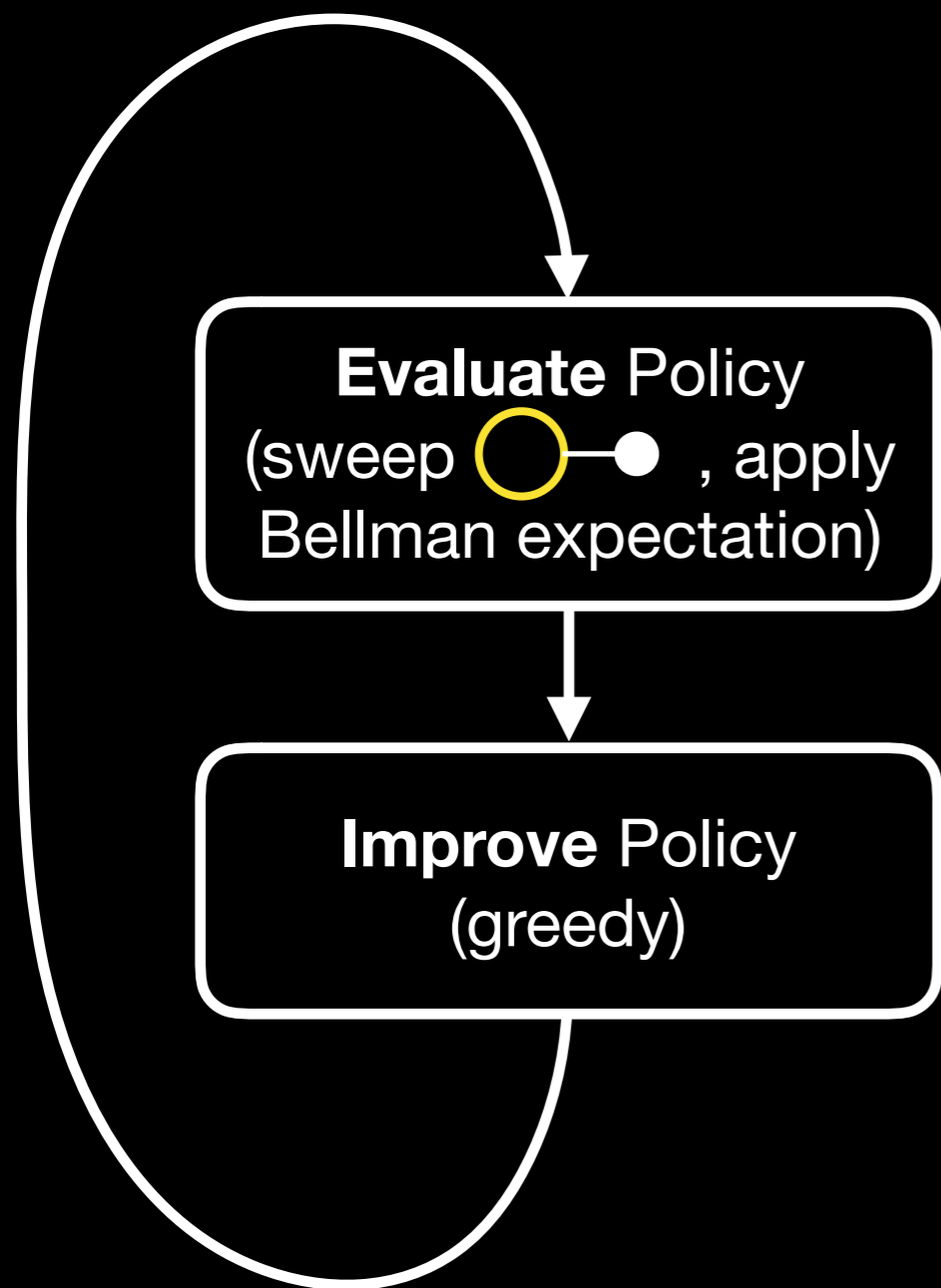


Dynamic Programming

...using Bellman equations as iterative updates



Policy Iteration



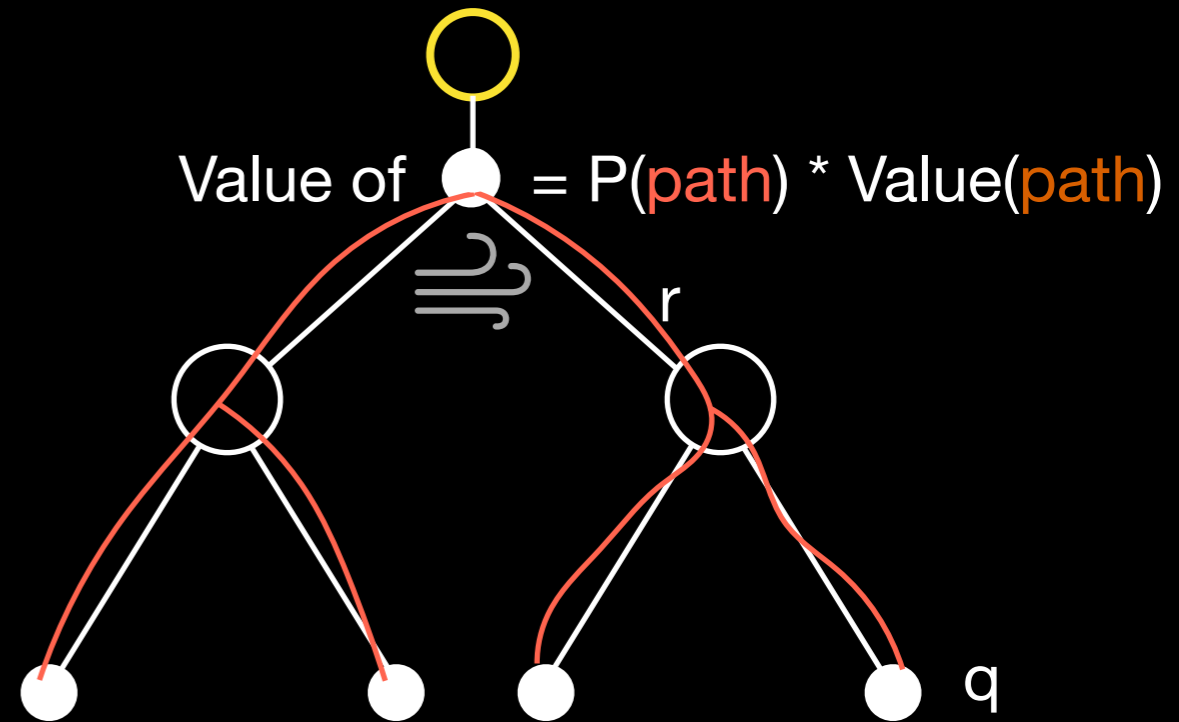
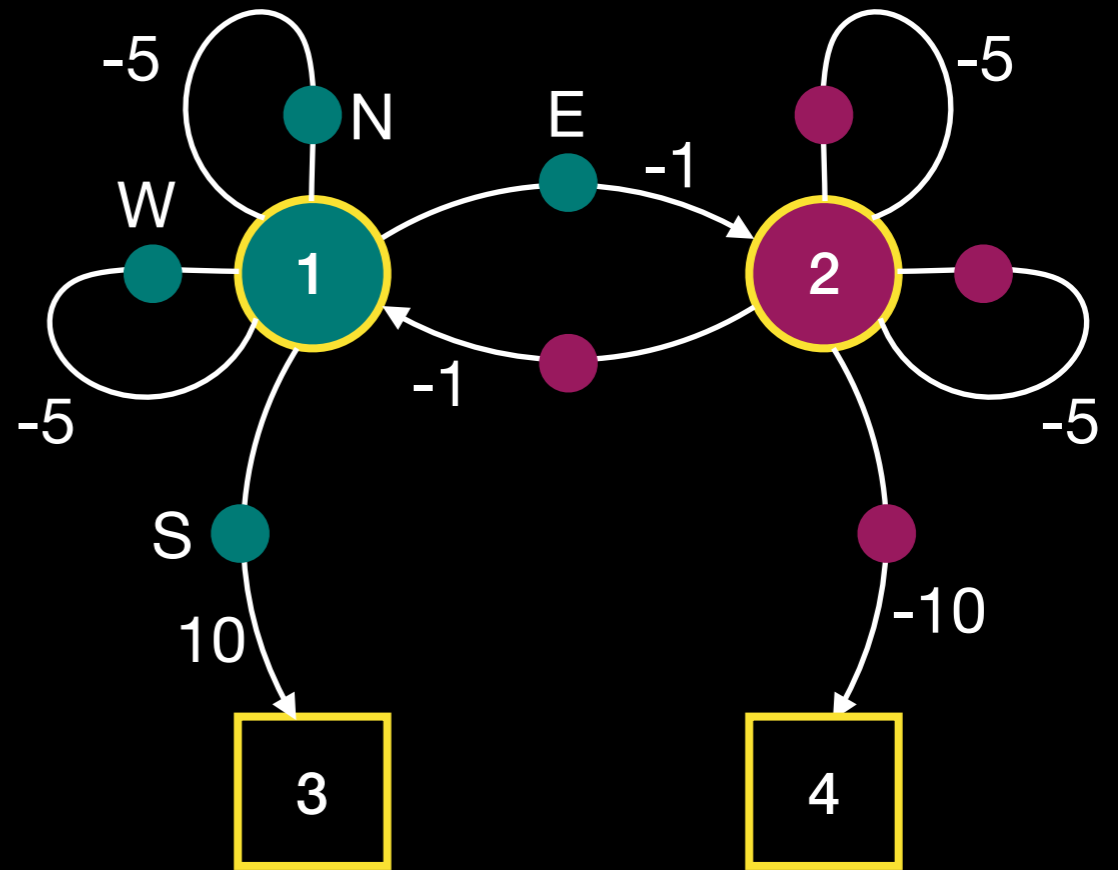
$$q_{\pi}(s, a) = r_s^a + \gamma \sum_{s'} P_{ss'}^a \sum_{a'} \pi(a' | s') q_{\pi}(s', a')$$

N: $-5 + 0.9 \cdot 0$ N: $-5 + 0.9 \cdot 0$
 E: $-1 + 0.9 \cdot 0$ E: $-5 + 0.9 \cdot 0$
 S: $10 + 0.9 \cdot 0$ S: $-10 + 0.9 \cdot 0$
 W: $-5 + 0.9 \cdot 0$ W: $-1 + 0.9 \cdot 0$

iteratively apply Bellman expectation equations in inner loop until values do not change much

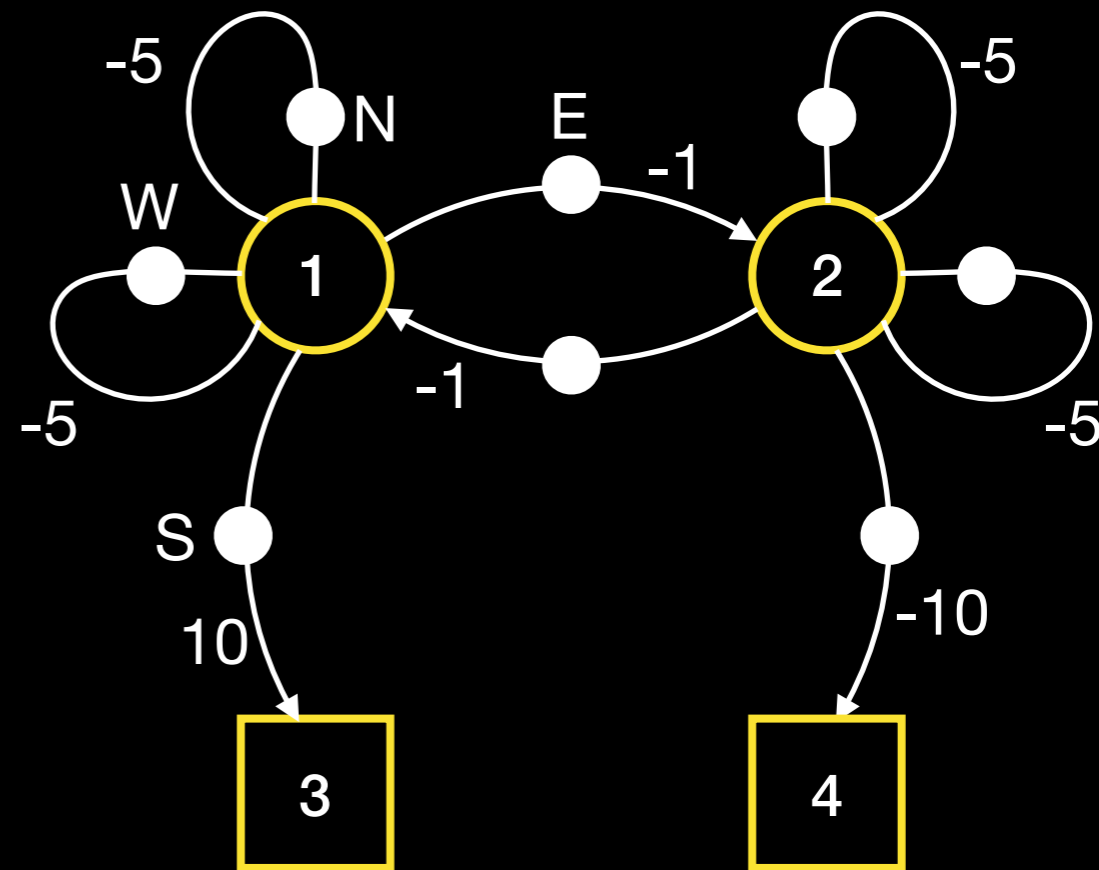
use greedy policy, given new values

$\pi(S|1): 1.0$ (greedy)
 $\pi(W|2): 1.0$ (greedy)



$$q_{\pi}(s, a) = r_s^a + \gamma \sum_{s'} P_{ss'}^a \sum_{a'} \pi(a' | s') q_{\pi}(s', a')$$

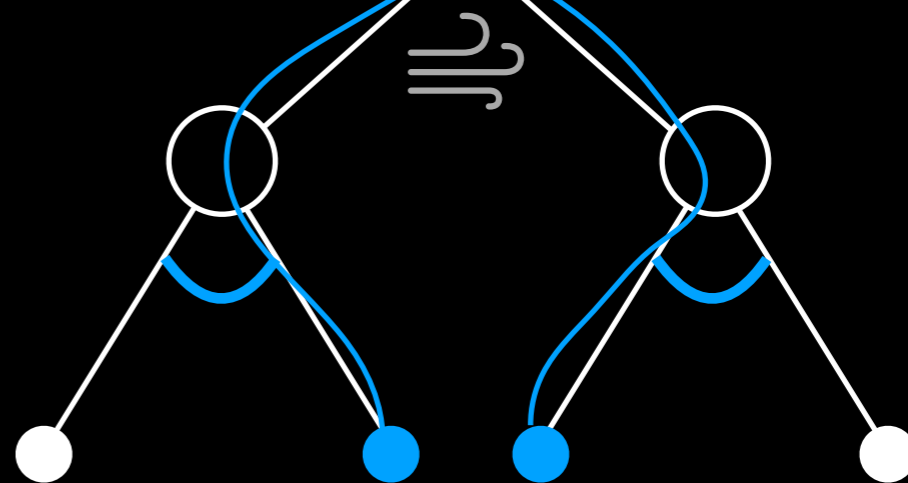
Value Iteration



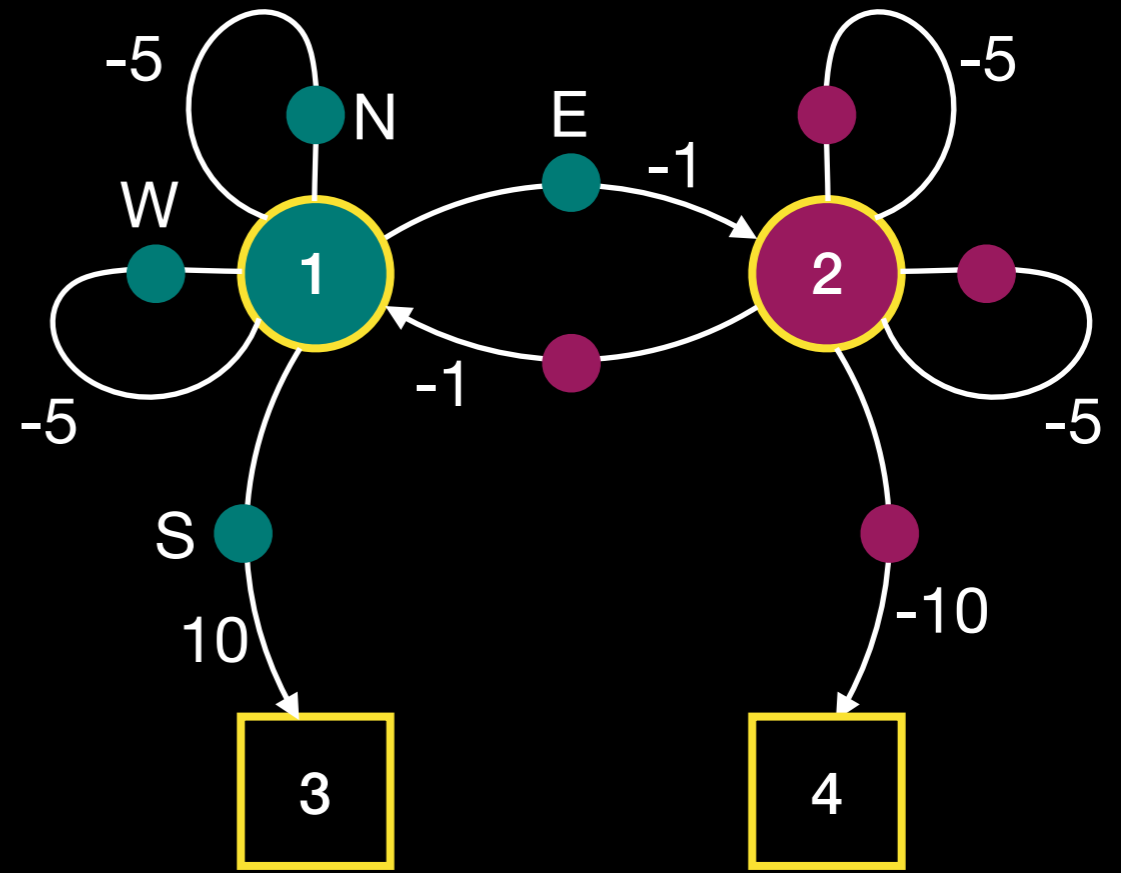
Find Optimal Value and Policy

(sweep , apply Bellman optimality)

Value of = $P(\text{path}) * \text{Value}(\text{path})$

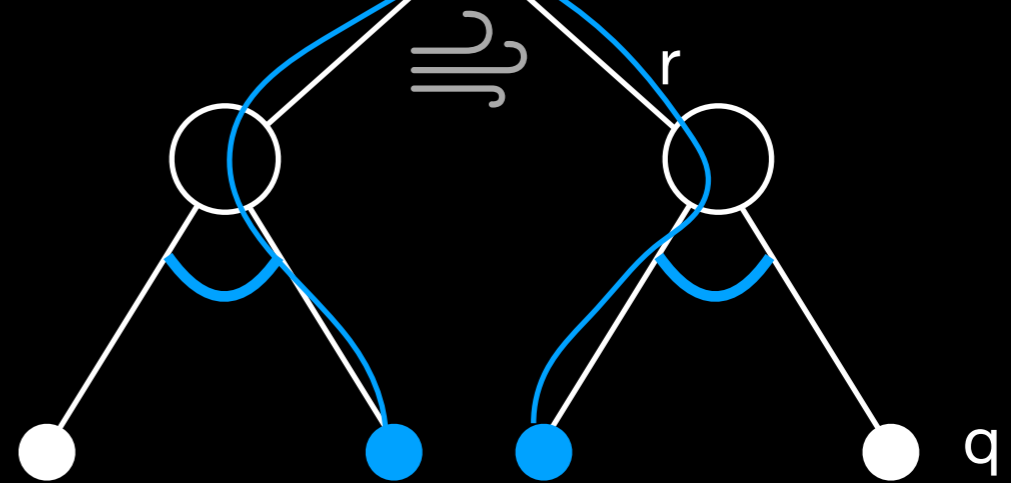


$$q_*(s, a) = r_s^a + \gamma \sum_{s'} P_{ss'}^a \max_{a'} q_*(s', a')$$



N: $-5 + 0.9 \cdot 0$	N: $-5 + 0.9 \cdot 0$
E: $-1 + 0.9 \cdot 0$	E: $-5 + 0.9 \cdot 0$
S: $10 + 0.9 \cdot 0$	S: $-10 + 0.9 \cdot 0$
W: $-5 + 0.9 \cdot 0$	W: $-1 + 0.9 \cdot 0$

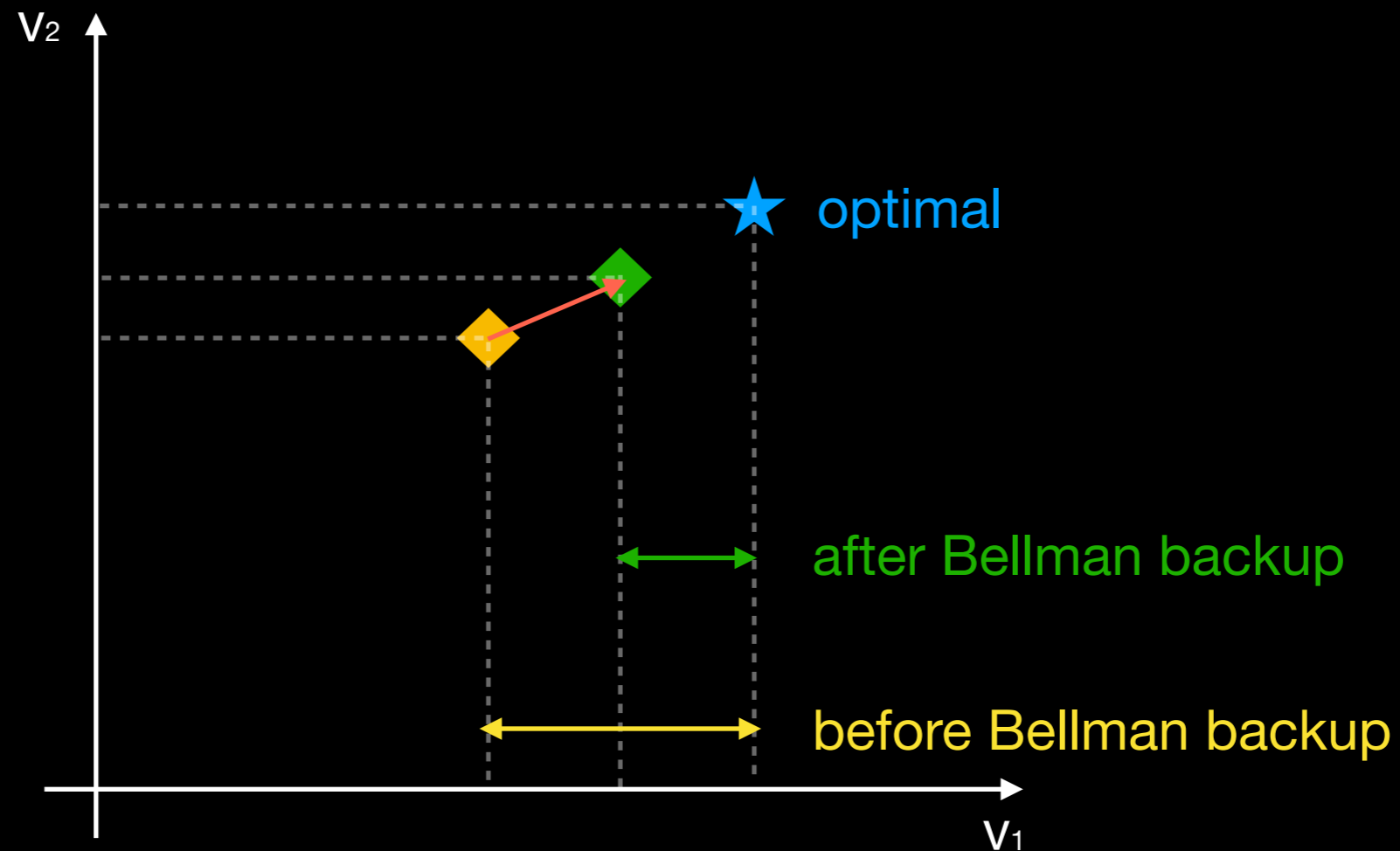
Value of $\bullet = P(\text{path}) * \text{Value}(\text{path})$



$$q_*(s, a) = r_s^a + \gamma \sum_{s'} P_{ss'}^a \max_{a'} q_*(s', a')$$

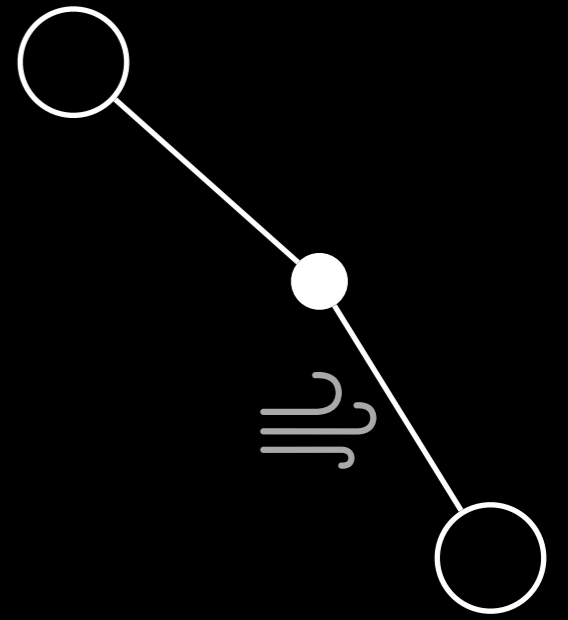
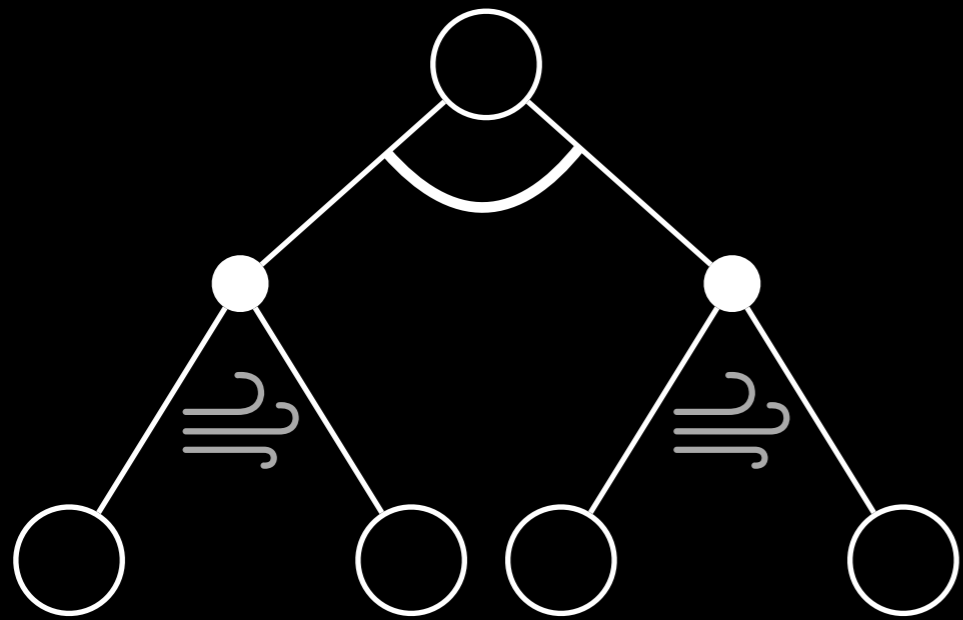
iteratively apply Bellman optimality equations until values do not change much

Bellman backups

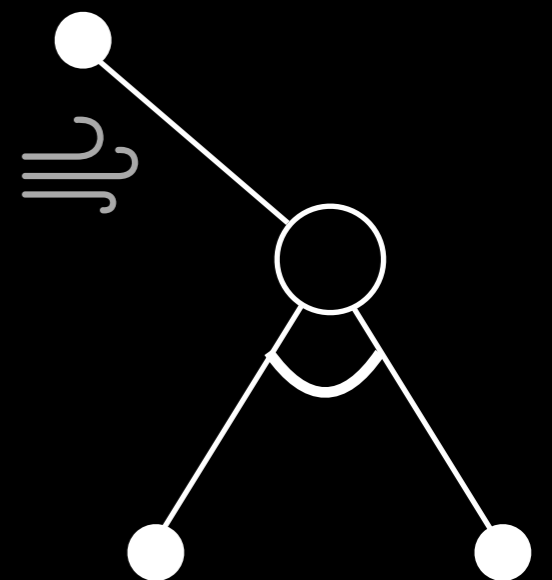
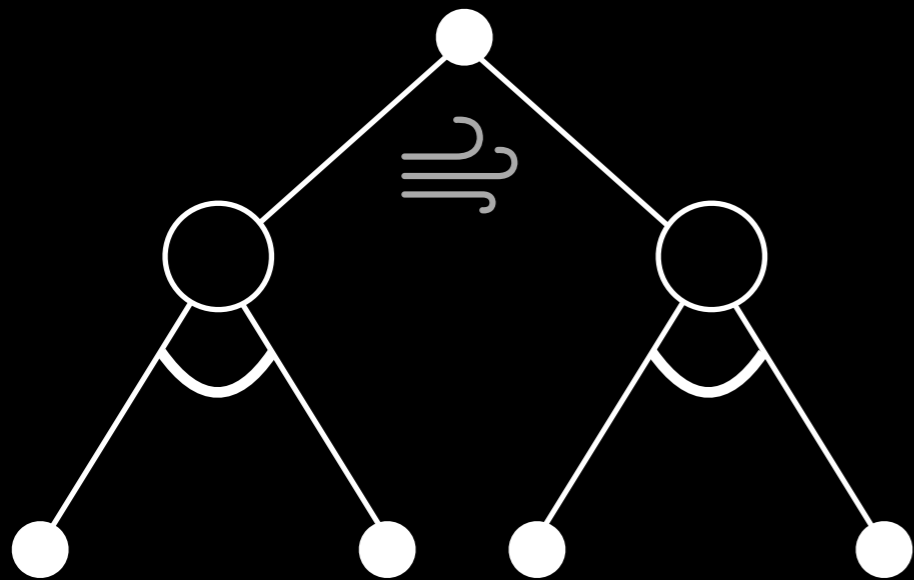


largest distance between values
decreases after **Bellman backups**

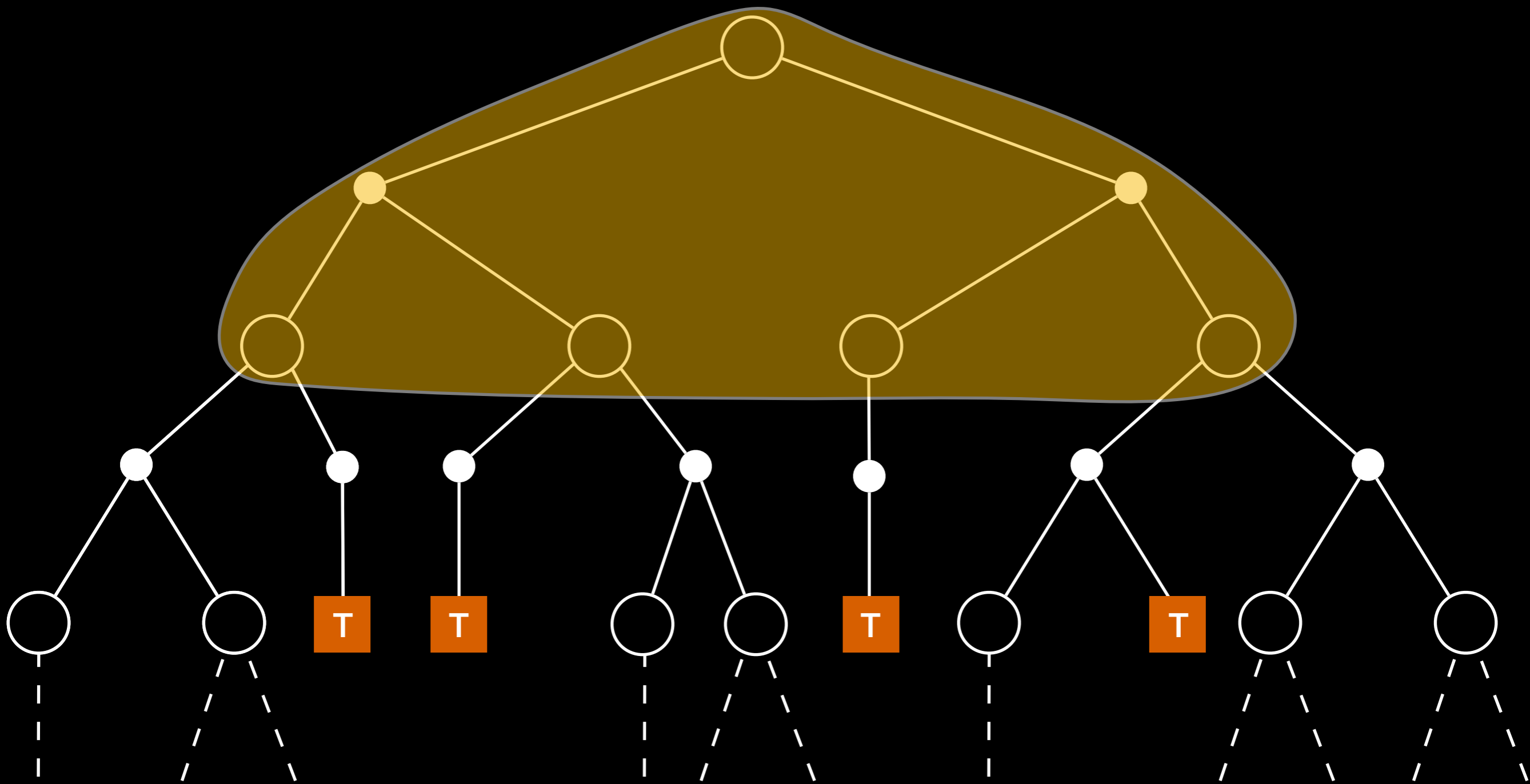
From DP to Learning



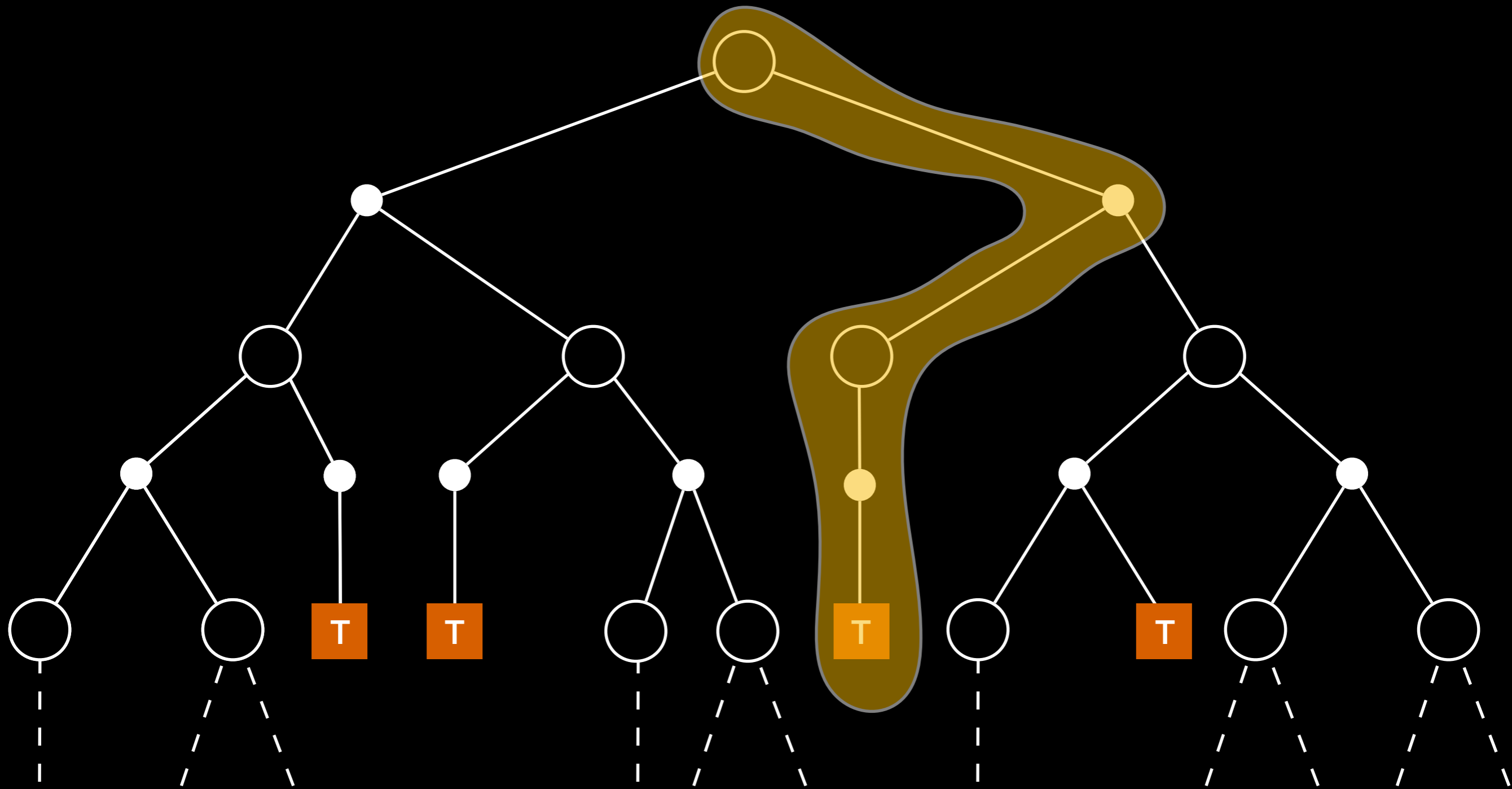
full-width backups
to **sample** backups



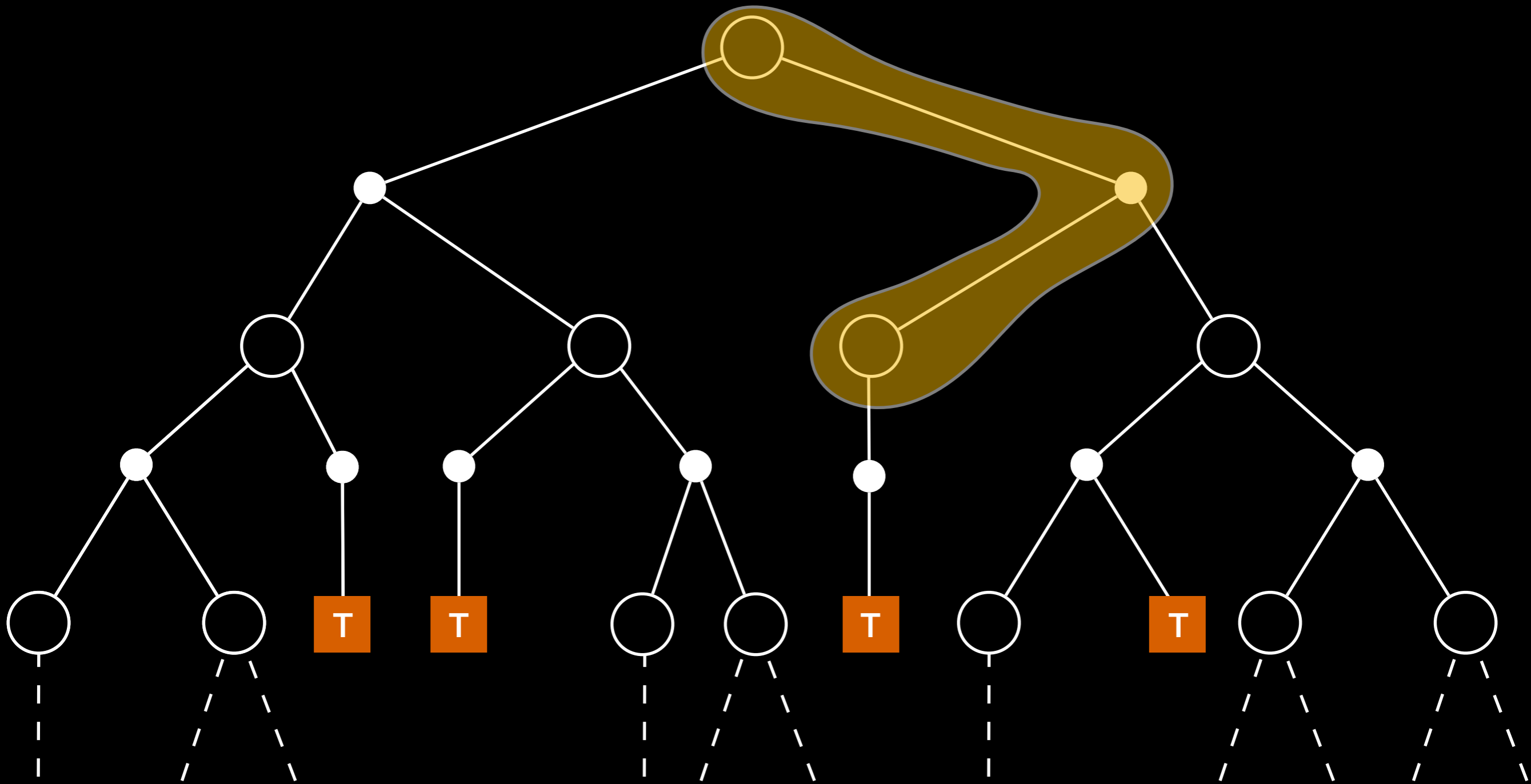
Full-width Backup



Backup with Sample Return



Backup with Guess



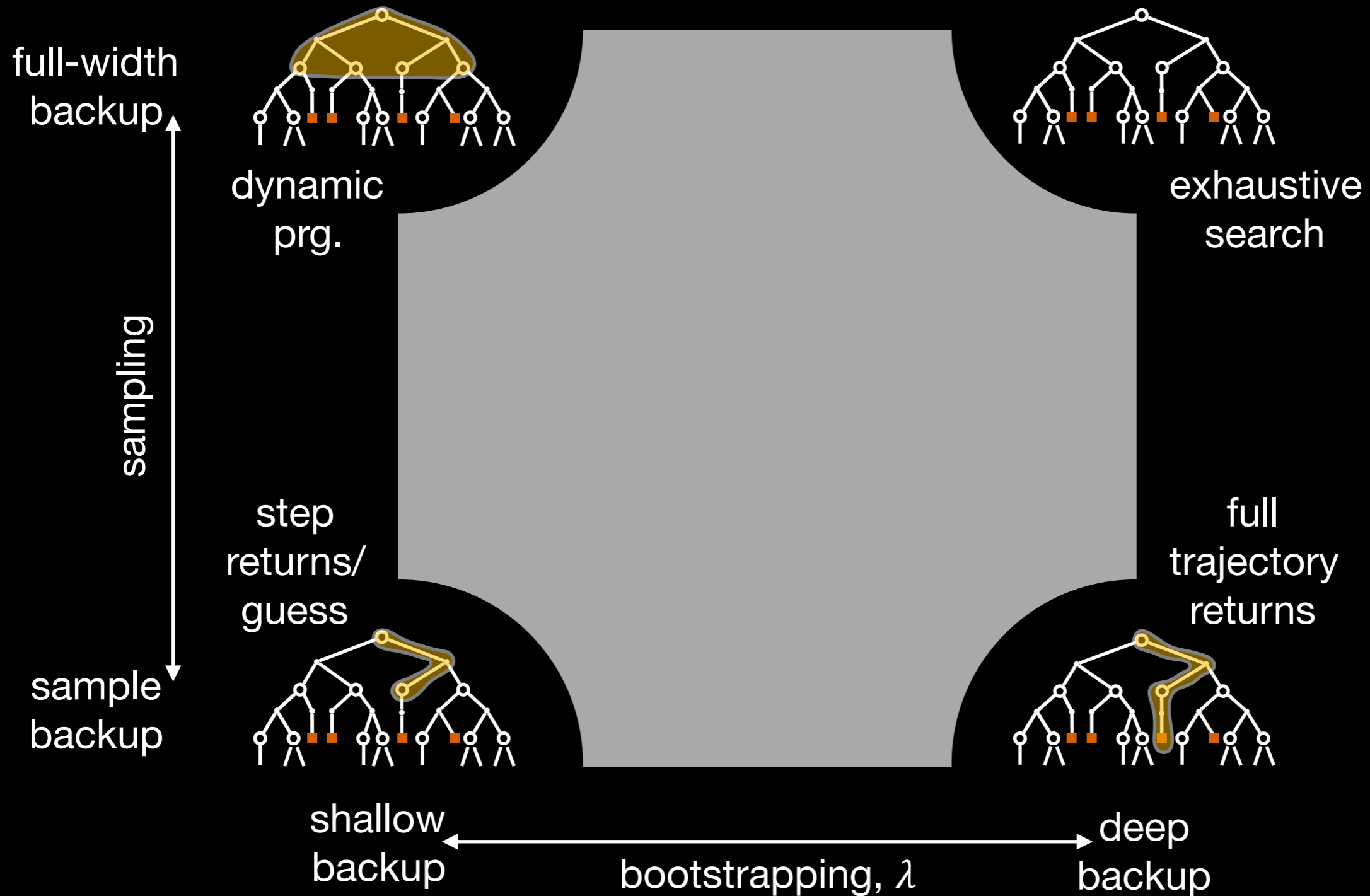
Incremental Updates

$$E\{R\} \approx \mu_k = \frac{1}{k} \sum_{\tau=1}^k R_{\tau} \quad \text{batched}$$

$$\mu_k = \mu_{k-1} + \frac{1}{k} (R_k - \mu_{k-1}) \quad \text{incremental}$$

$$\mu_k = \mu_{k-1} + \alpha (R_k - \mu_{k-1}) \quad \begin{array}{l} \text{running} \\ \text{(saw this in} \\ \text{Q-learning!)} \end{array}$$

Sample and Bootstrap



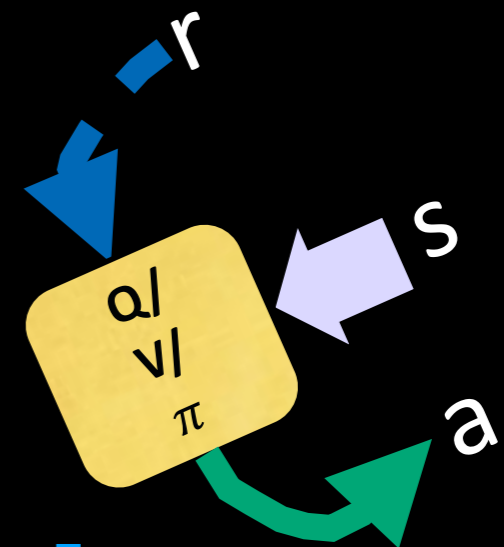
It all comes down to:

estimating returns

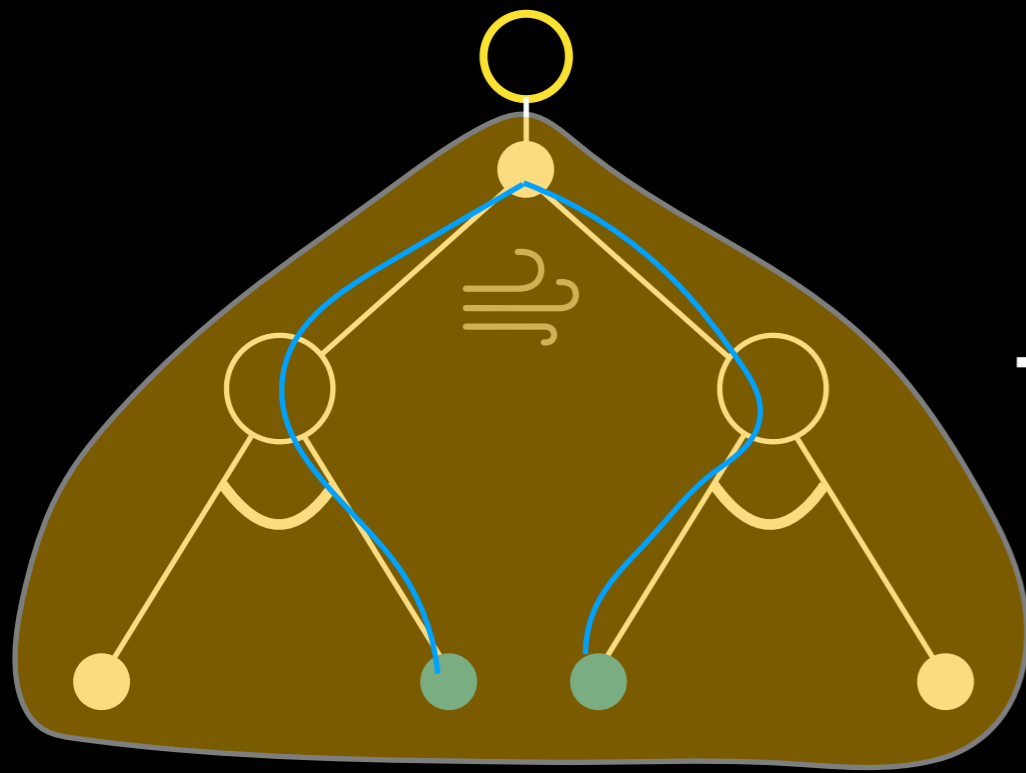
optimising

towards achieving

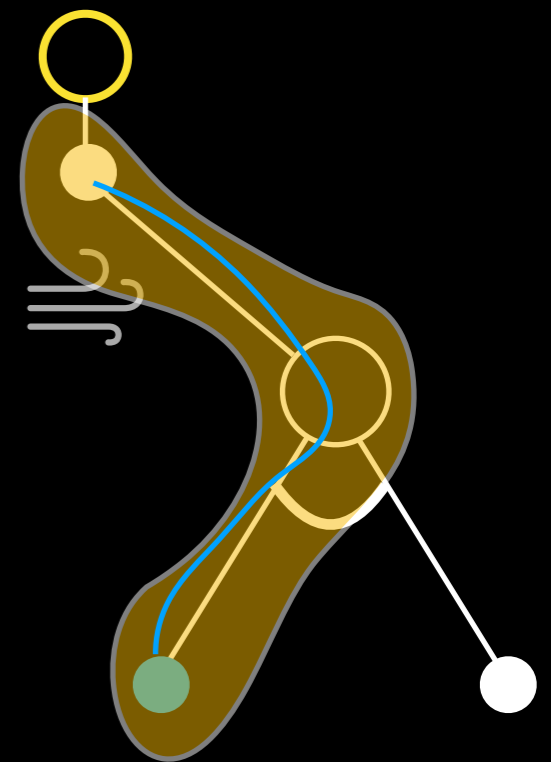
returns



Q-learning

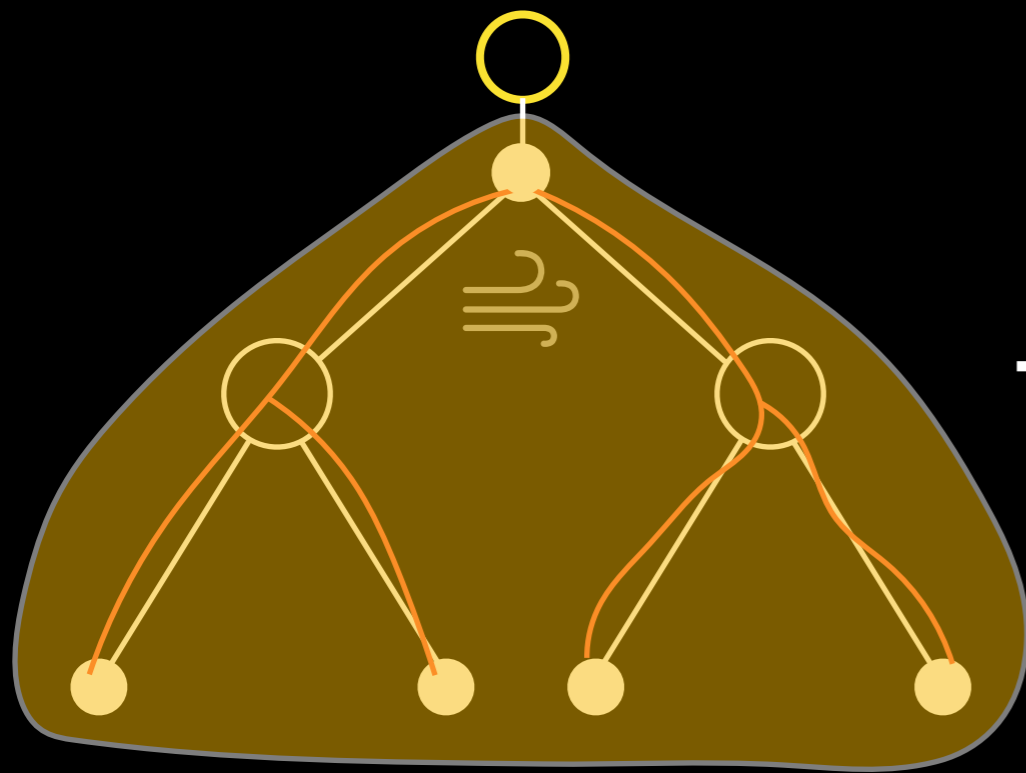


full-width backups
to **sample** backups

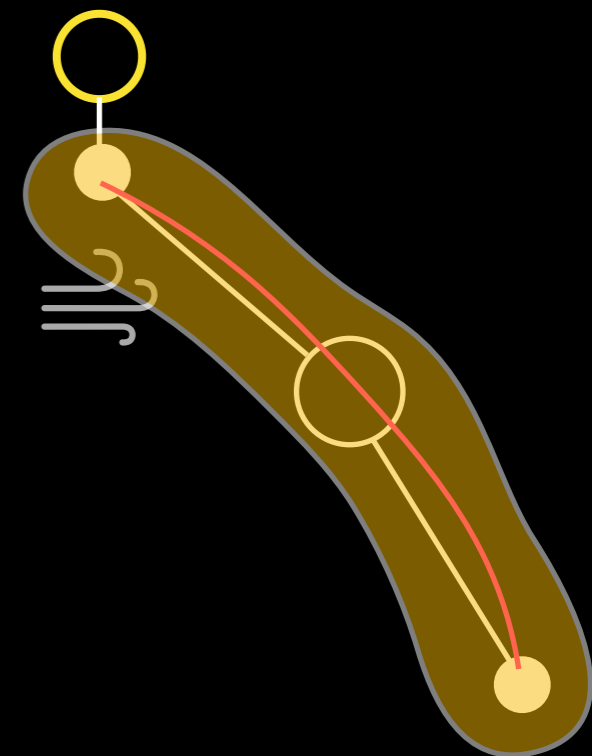


target policy
optimal

SARSA



full-width backups
to **sample** backups



target policy
same as
behaviour policy

scaling up RL with
function approximation

Approximate Q-learning

e.g. linear approximation

$$Q_{\theta}(s, a) = \theta_0 f_0(s, a) + \theta_1 f_1(s, a) + \dots + \theta_n f_n(s, a)$$

$$Q_{\text{target}} = (r_s^a + \gamma \max_{a'} Q(s', a'))$$

$$\theta \leftarrow \theta - \alpha \nabla_{\theta} \frac{1}{2} \left(Q_{\text{target}} - Q_{\theta}(s, a) \right)^2$$

gradient updates equivalent to tabular Q updates

Say $\theta \in \mathbb{R}^{|S| \times |A|}$, so $Q_\theta(s, a) = \theta_{sa}$

$$Q_{\text{target}} = r_s^a + \gamma \max_{a'} Q(s', a')$$

$$\theta_{sa} \leftarrow \theta_{sa} - \alpha \nabla_{\theta_{sa}} \frac{1}{2} (Q_{\text{target}} - \theta_{sa})^2$$

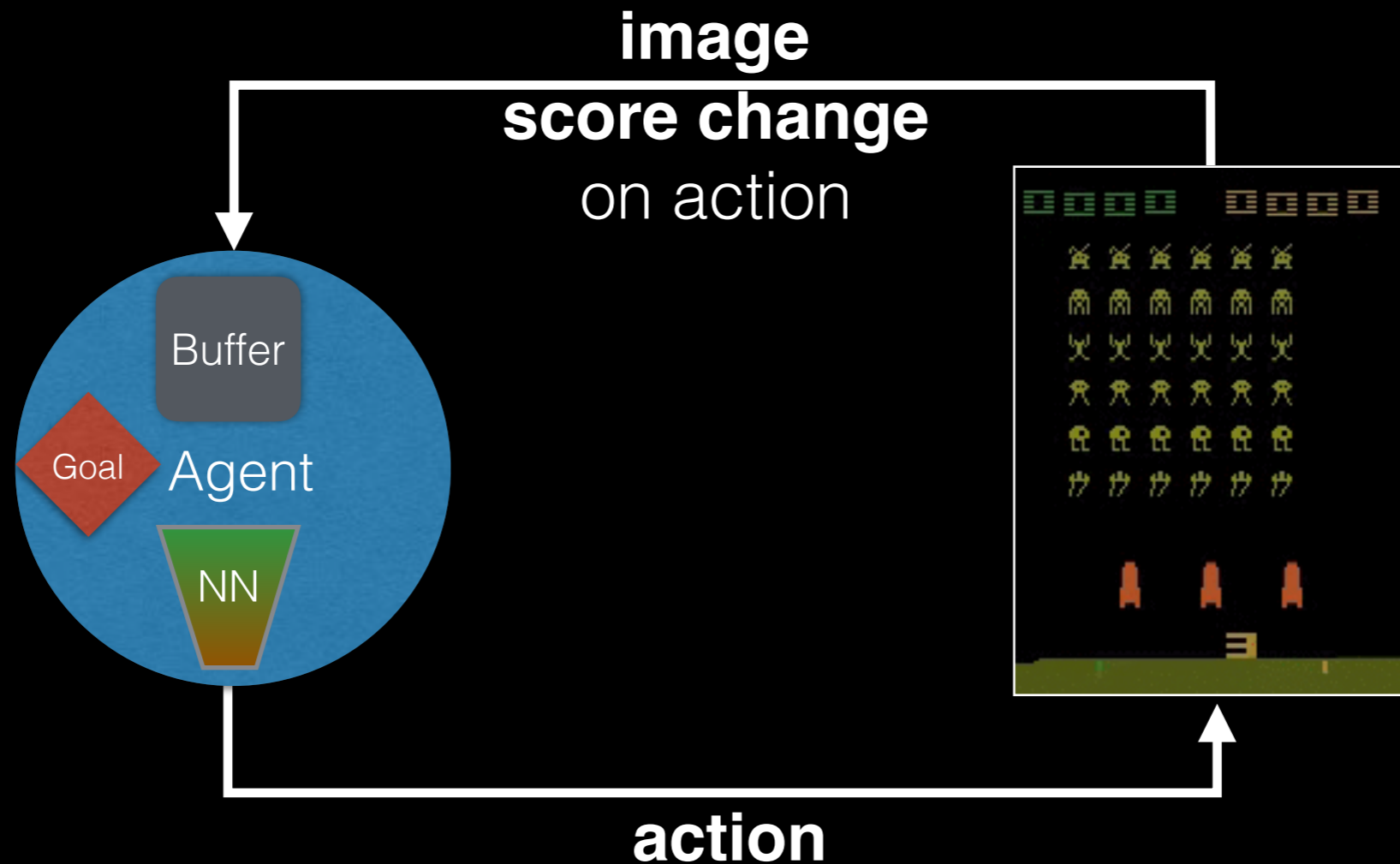
$$\theta_{sa} \leftarrow \theta_{sa} - \alpha (-Q_{\text{target}} + \theta_{sa})$$

$$\theta_{sa} \leftarrow \theta_{sa} + \alpha (Q_{\text{target}} - \theta_{sa})$$

$$\theta_{sa} \leftarrow (1 - \alpha) \theta_{sa} + \alpha Q_{\text{target}}$$

tabular
equivalent

DQN



Human-level control through deep reinforcement learning,
Mnih et. al., Nature 518, Feb 2015

human level game control

- **pixel** input
- **18 joystick/button positions** output
- **change in game score** as feedback
- **convolutional net representing Q**
- **backpropagation** for training!

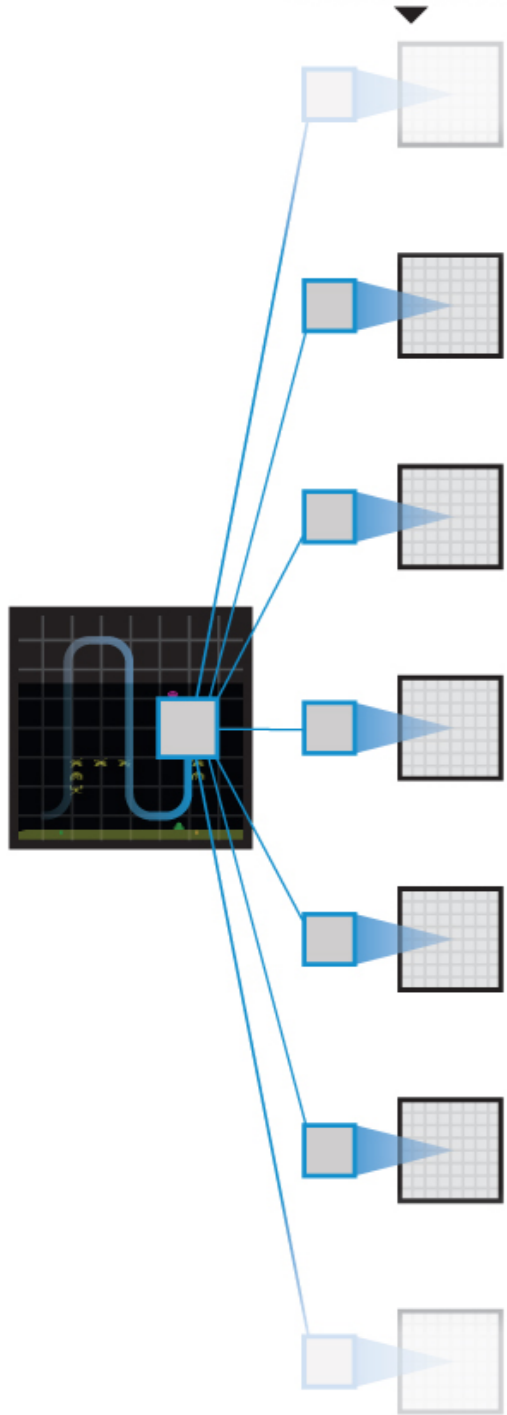
Human-level control through deep reinforcement learning,

Mnih et. al., Nature 518, Feb 2015

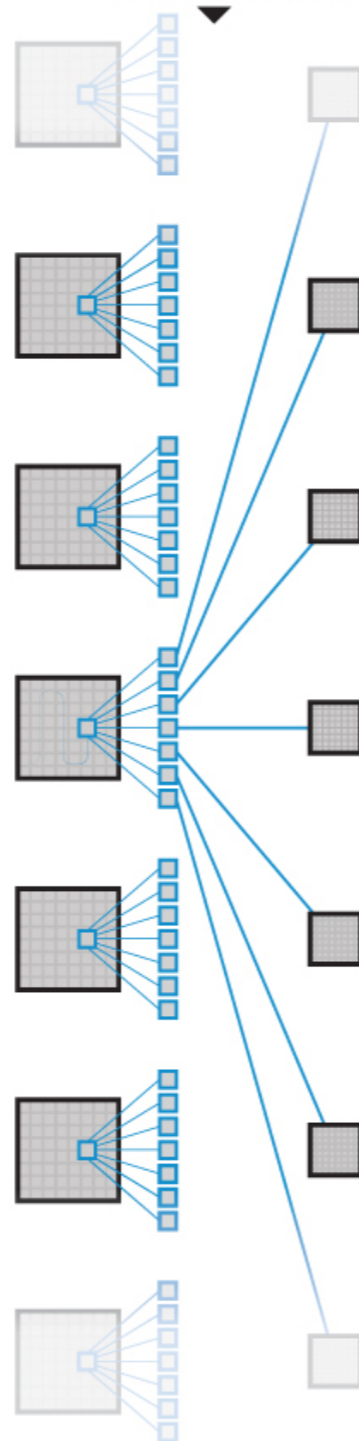
<http://www.nature.com/nature/journal/v518/n7540/full/nature14236.html>

neural network

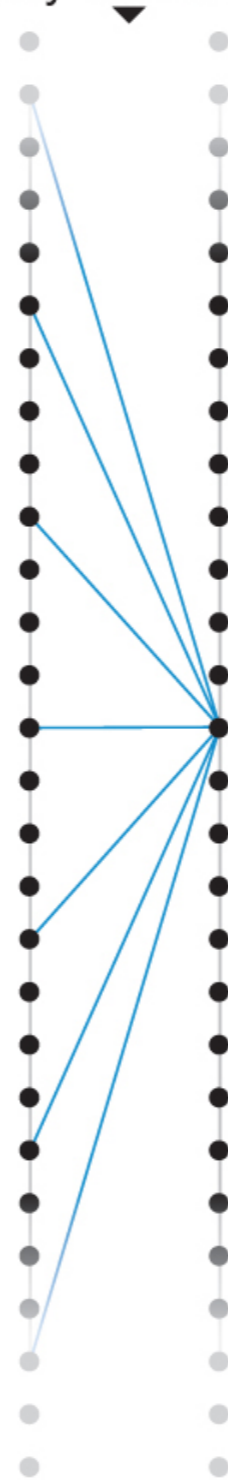
Convolution



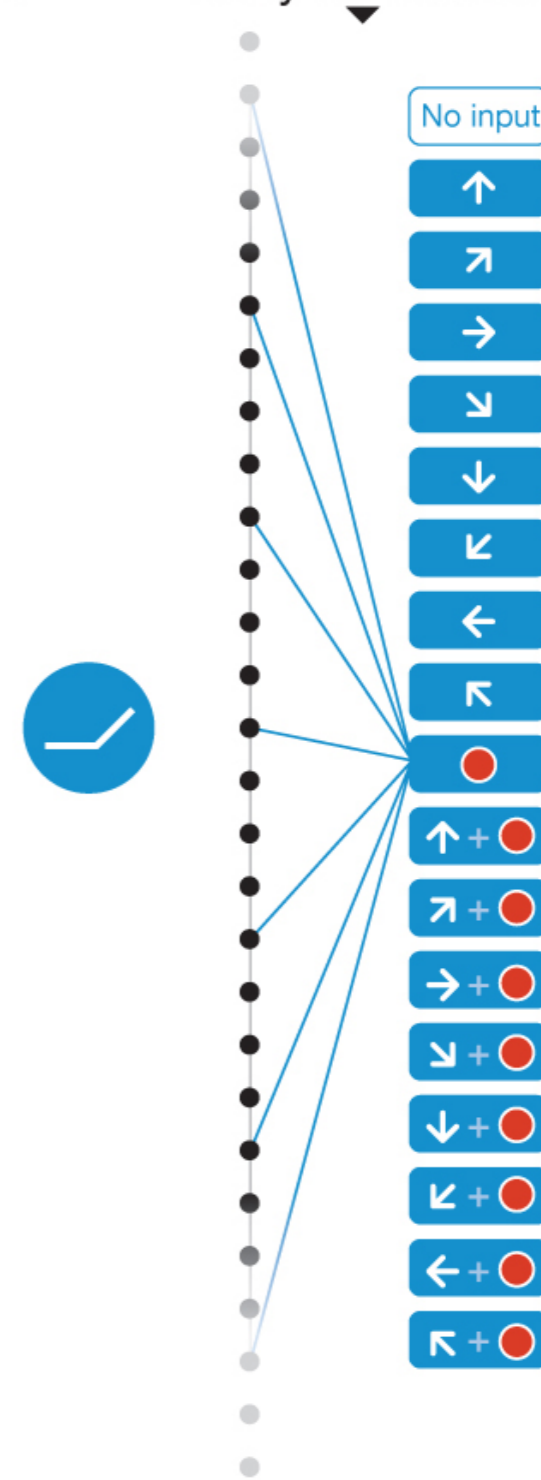
Convolution



Fully connected



Fully connected



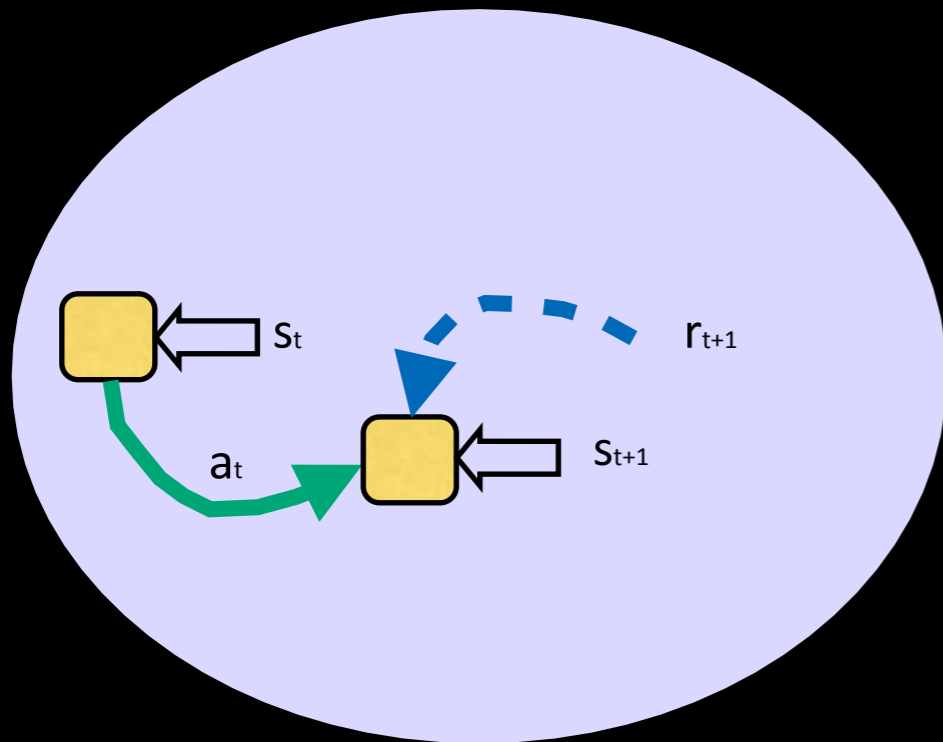
backpropagation

What is the **target** against which to minimise error?

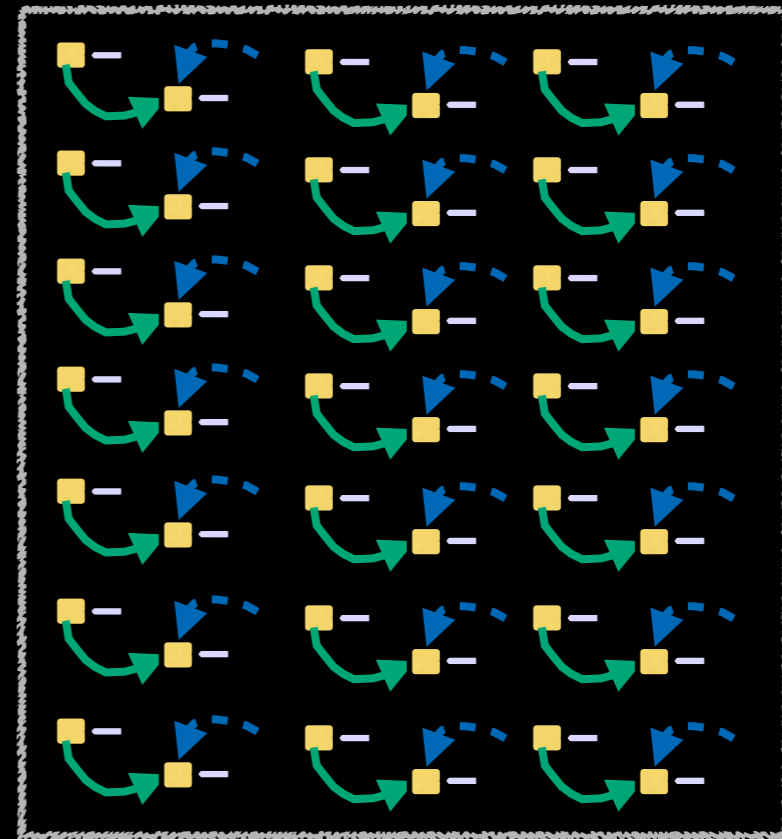
$$\mathcal{L}(w) = \mathbb{E} \left[\left(\underbrace{r + \gamma \max_{a'} Q(s', a', w)}_{\text{target}} - Q(s, a, w) \right)^2 \right]$$

$$\frac{\partial \mathcal{L}(w)}{\partial w} = \mathbb{E} \left[\left(r + \gamma \max_{a'} Q(s', a', w) - Q(s, a, w) \right) \frac{\partial Q(s, a, w)}{\partial w} \right]$$

experience replay buffer



save transition in
memory

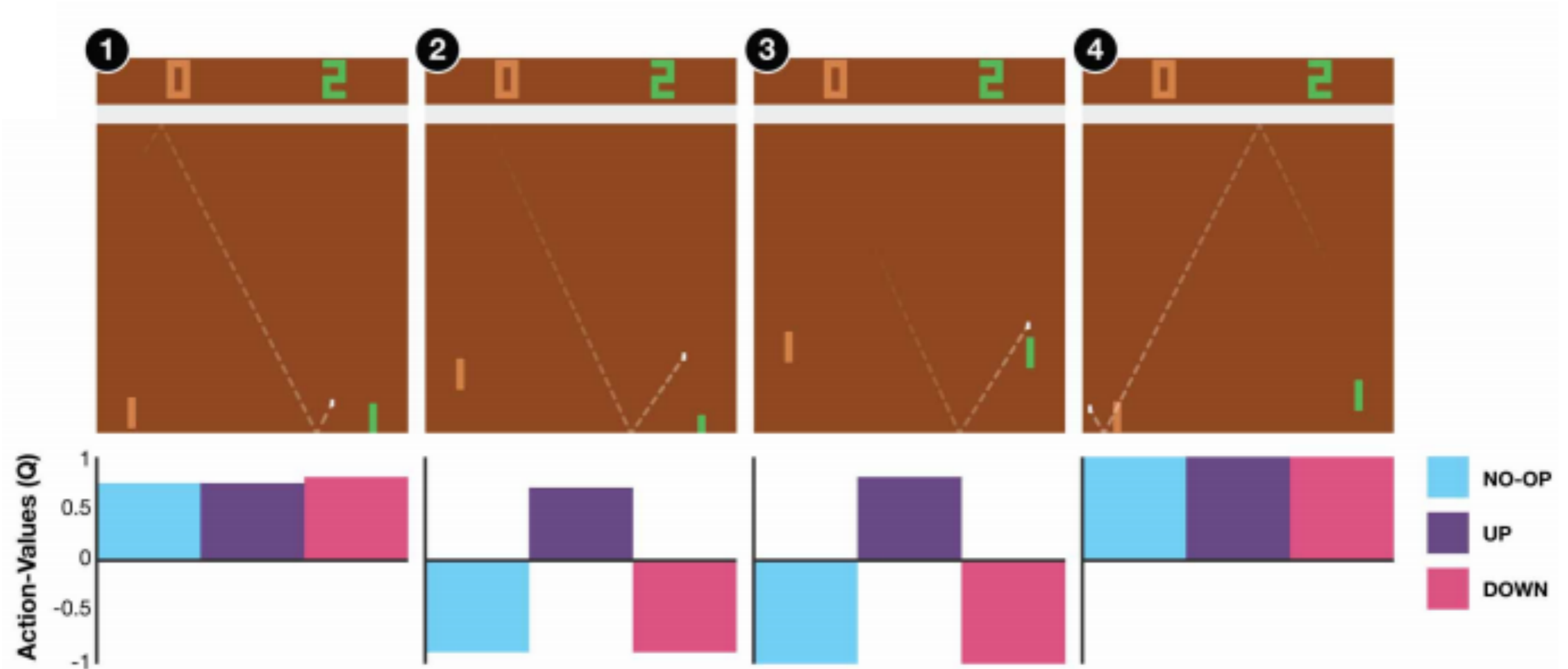


randomly **sample**
from memory
for training
= i.i.d

freeze
target

freeze

$$\left(r + \gamma \max_{a'} Q(s', a', \mathbf{w}^-) - Q(s, a, \mathbf{w}) \right)^2$$



<https://storage.googleapis.com/deepmind-media/dqn/DQNNaturePaper.pdf>

however
training is

SLOOOO...W

parallelise...

Parallel Asynchronous Training

value and **policy** based methods



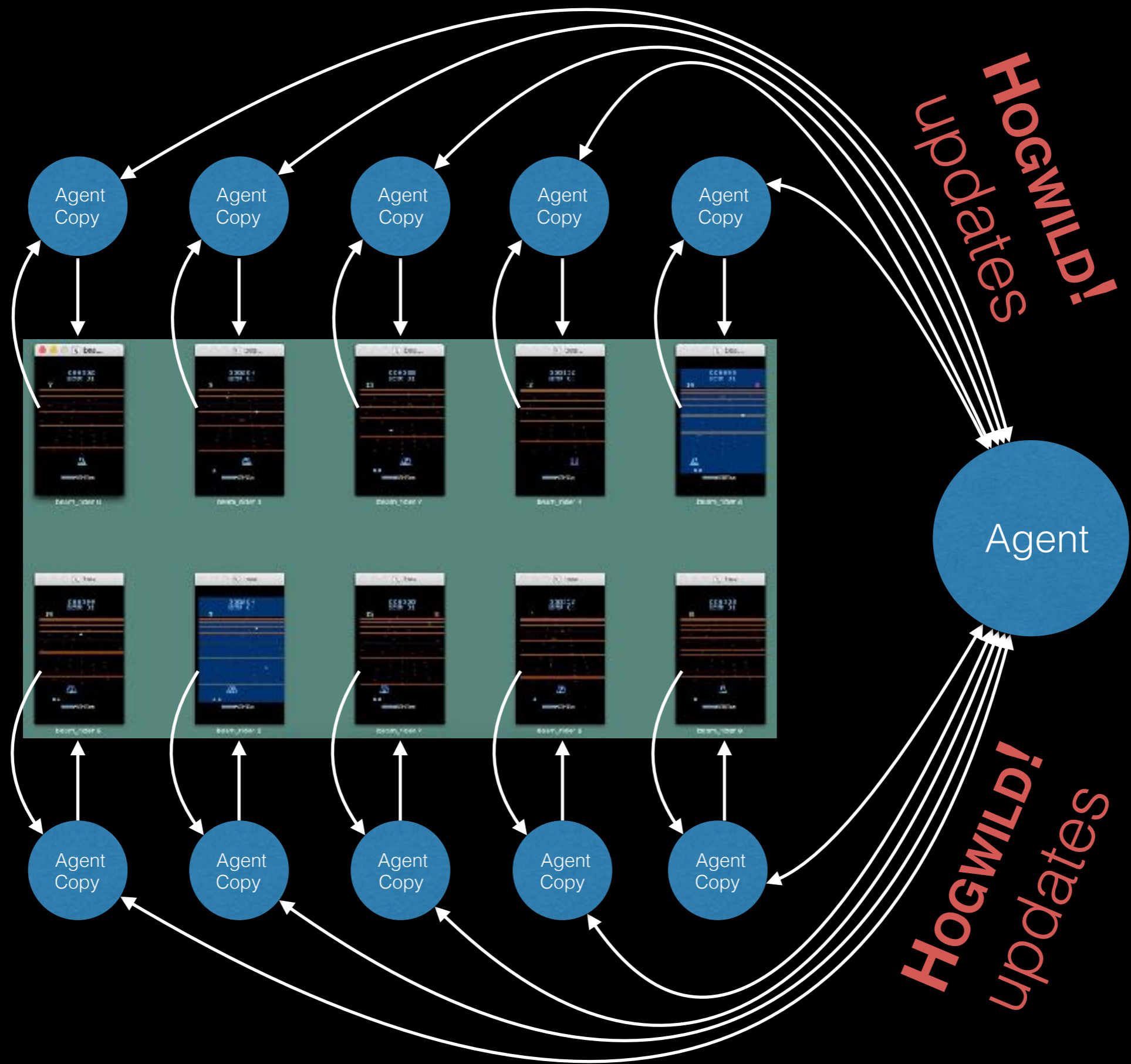
<https://youtu.be/0xo1Ldx3L5Q>

parallel
agents

shared
parameters

lock-free
updates

parallel learners



params shared

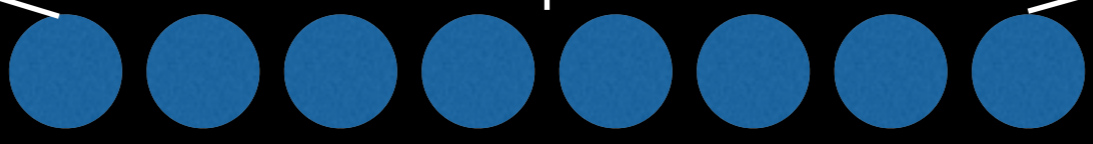
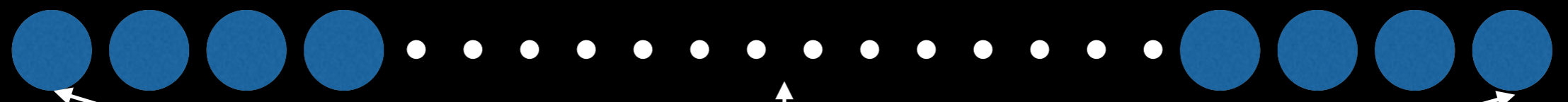
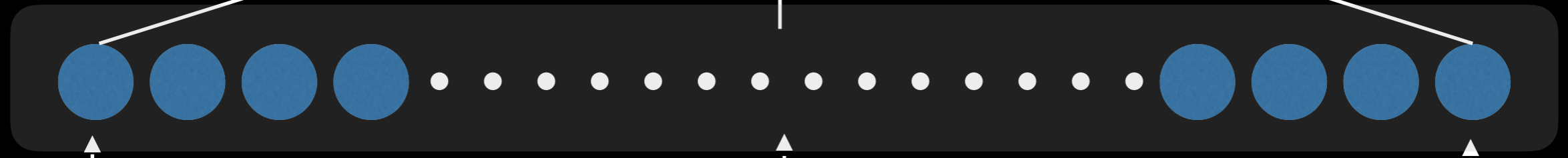
Policy Based

π

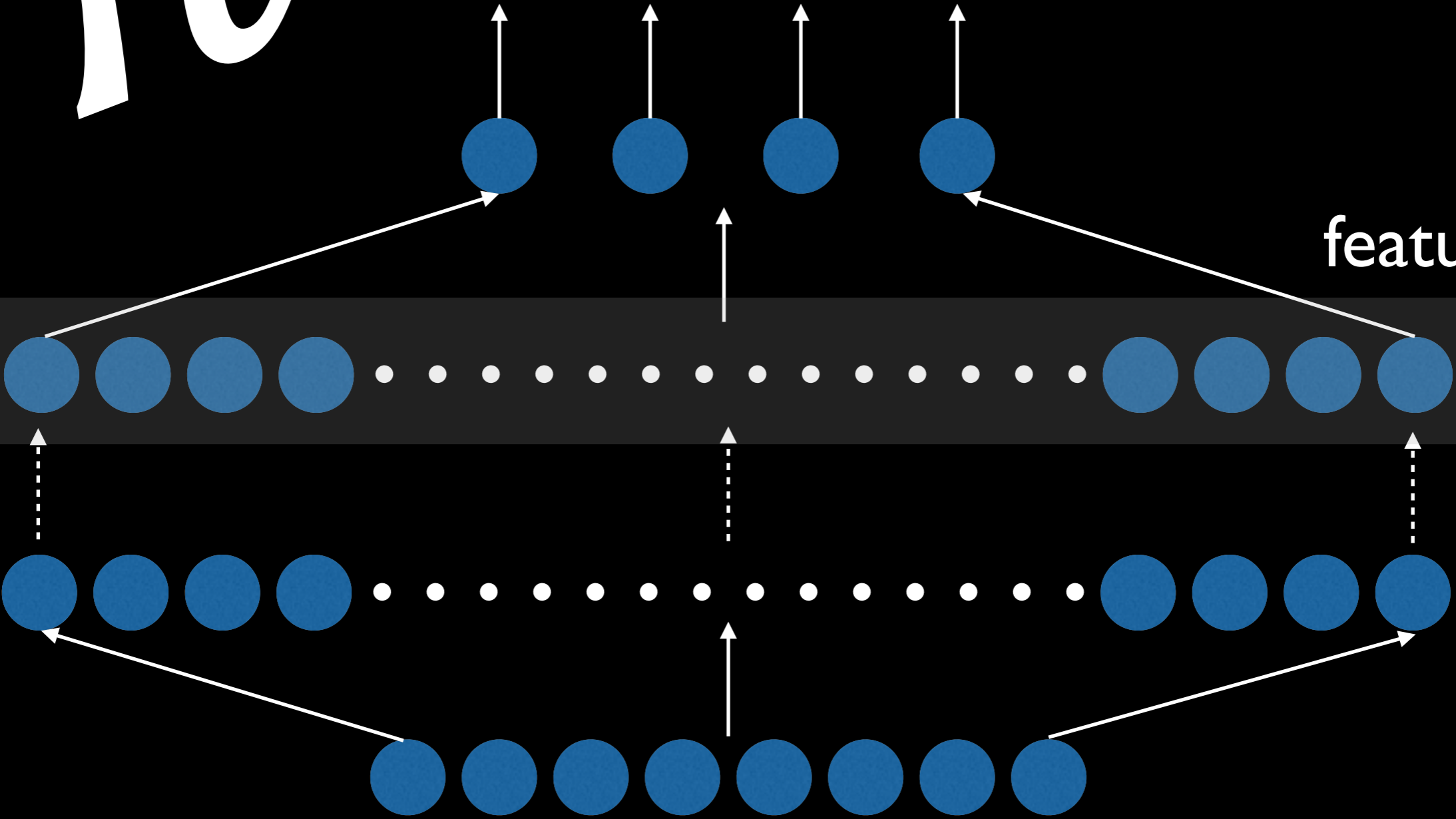
policy $\pi(a|s)$

$\pi(\text{north}|s)$ $\pi(\text{south}|s)$ $\pi(\text{east}|s)$ $\pi(\text{west}|s)$

features

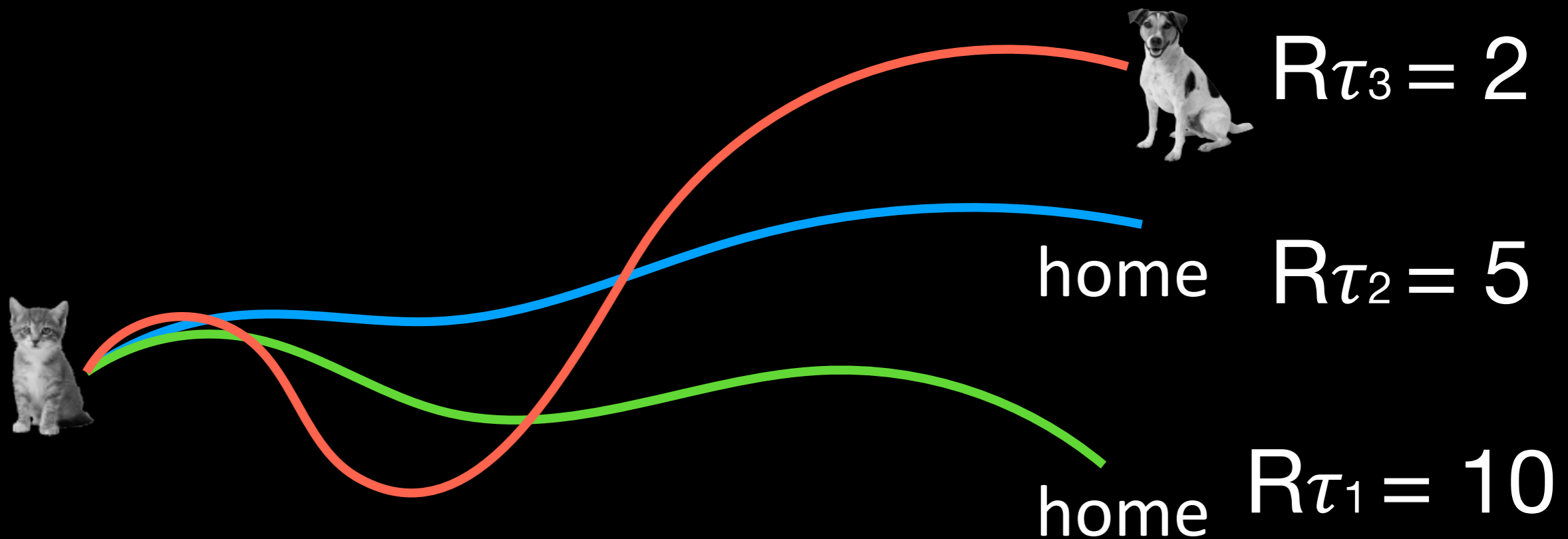


state s



Intuition

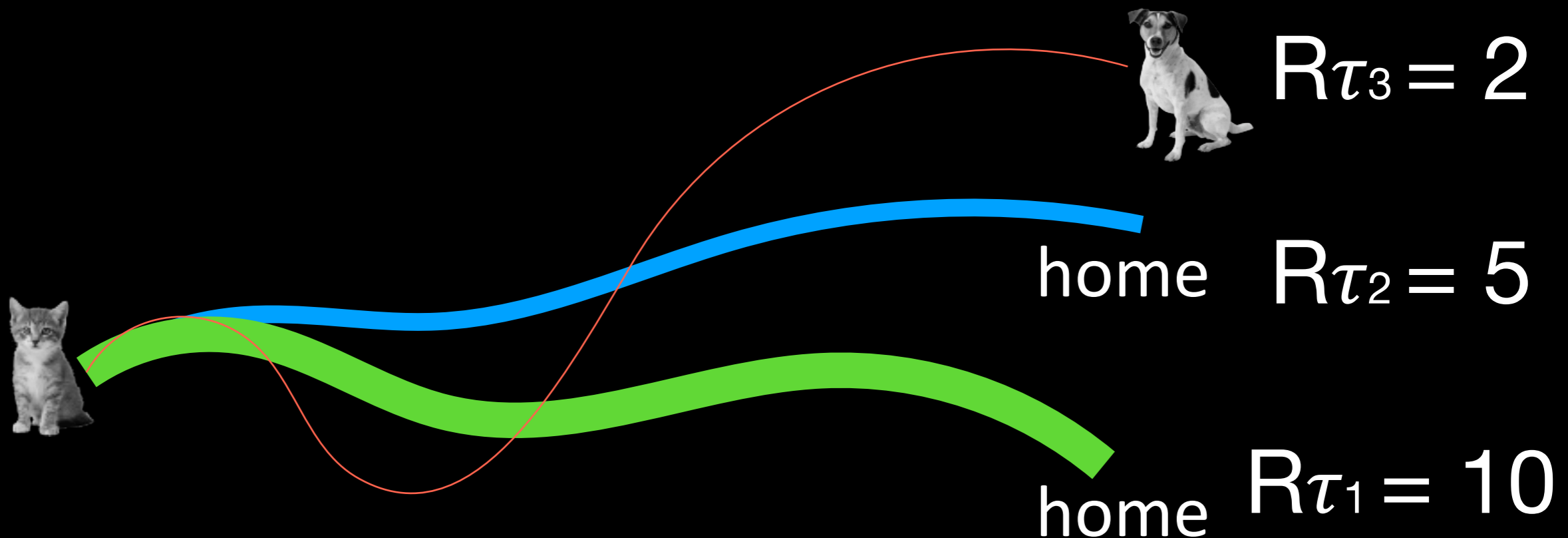
$\tau : s_1, a_1, r_1^1, s_2, a_2, r_2^2, \dots, s_{H-1}, a_{H-1}, r_{H-1}^{H-1}$



Intuition

$\pi(\mathbf{a}|\mathbf{s})$ along path with high return higher

$$\tau : s_1, a_1, r_1^1, s_2, a_2, r_2^2, \dots, s_{H-1}, a_{H-1}, r_{H-1}^{H-1}$$



probabilities are relative

Revisiting the Objective

$$\tau : s_1, a_1, r_1^1, s_2, a_2, r_2^2, \dots, s_{H-1}, a_{H-1}, r_{H-1}^{H-1}$$

$$\max_{\theta} \mathbb{E}_{\tau} \left\{ \sum_{t=0}^{H-1} r_{s_t}^{a_t} \mid \pi_{\theta} \right\}$$



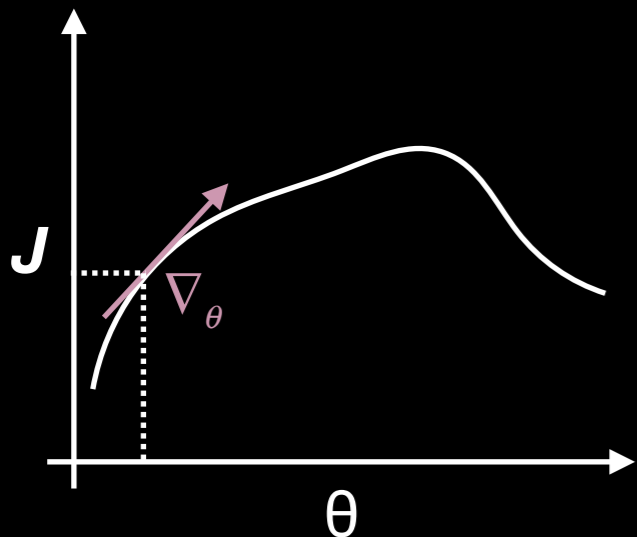
$$\max_{\theta} J(\theta) = \max_{\theta} \sum_{\tau} P(\tau \mid \theta) R(\tau)$$

Samples \rightarrow Gradient

$$J(\theta) = \sum_{\tau} P(\tau | \theta) R(\tau)$$

$$\max_{\theta} J(\theta)$$

$$\theta \leftarrow \theta + \nabla_{\theta} J(\theta)$$



gradient
via
sampling

$$\nabla_{\theta} J(\theta) = \nabla_{\theta} \sum_{\tau} \underline{P(\tau | \theta)} R(\tau)$$

$$= \sum_{\tau} \nabla_{\theta} P(\tau | \theta) R(\tau)$$

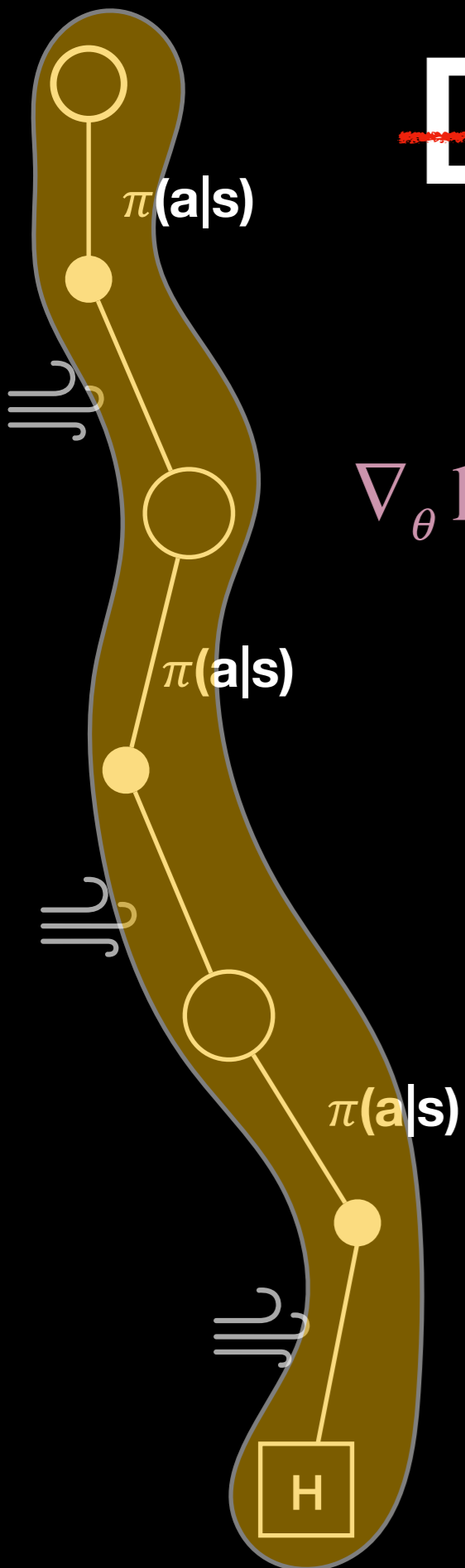
$$= \sum_{\tau} \frac{P(\tau | \theta)}{P(\tau | \theta)} \nabla_{\theta} P(\tau | \theta) R(\tau)$$

$$= \sum_{\tau} P(\tau | \theta) \frac{\nabla_{\theta} P(\tau | \theta)}{P(\tau | \theta)} R(\tau)$$

$$= \sum_{\tau} \underline{P(\tau | \theta)} \nabla_{\theta} \log P(\tau | \theta) R(\tau)$$

$$\nabla_{\theta} J(\theta) \approx \frac{1}{m} \sum_{i=1}^m \nabla_{\theta} \log P(\tau^{(i)} | \theta) R(\tau^{(i)})$$

~~Dynamics Model~~



$$\nabla_{\theta} \log P(\tau | \theta) = \nabla_{\theta} \log \left[\prod_{t=0}^{H-1} \overbrace{P(s_{t+1} | s_t, a_t)}^{\text{dynamics model}} \cdot \overbrace{\pi_{\theta}(a_t | s_t)}^{\text{policy}} \right]$$

$$\cong \nabla_{\theta} \left[\sum_{t=0}^{H-1} \log P(s_{t+1} | s_t, a_t) + \sum_{t=0}^{H-1} \log \pi_{\theta}(a_t | s_t) \right]$$

$$= \nabla_{\theta} \sum_{t=0}^{H-1} \log \pi_{\theta}(a_t | s_t) = \sum_{t=0}^{H-1} \nabla_{\theta} \log \pi_{\theta}(a_t | s_t)$$

$$\nabla_{\theta} J(\theta) \approx \frac{1}{m} \sum_{i=1}^m \left(\sum_{t=0}^{H-1} \nabla_{\theta} \log \pi_{\theta}(a_t^{(i)} | s_t^{(i)}) \right) R(\tau^{(i)})$$

$$\nabla_{\theta} J(\theta) \approx \frac{1}{m} \sum_{i=1}^m \left(\sum_{t=0}^{H-1} \nabla_{\theta} \log \pi_{\theta}(a_t^{(i)} | s_t^{(i)}) \right) R(\tau^{(i)})$$

For each action a_t in state s_t
during each trajectory m

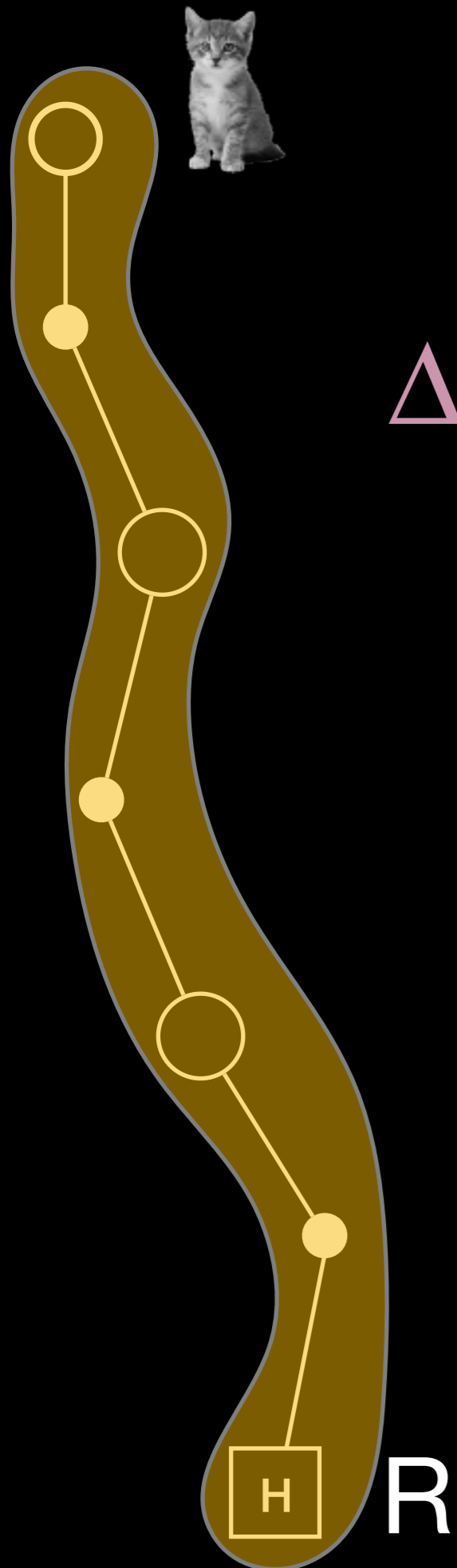
$$\underline{\nabla_{\theta} \log \pi_{\theta}(a_t | s_t)} \underline{R(\tau)}$$

$\Delta\theta$ to increase $\pi_{\theta}(a_t | s_t)$ x



Noisy Gradient

$\Delta\theta$ to increase $\pi_{\theta}(a_t | s_t) \times$



R



R



R



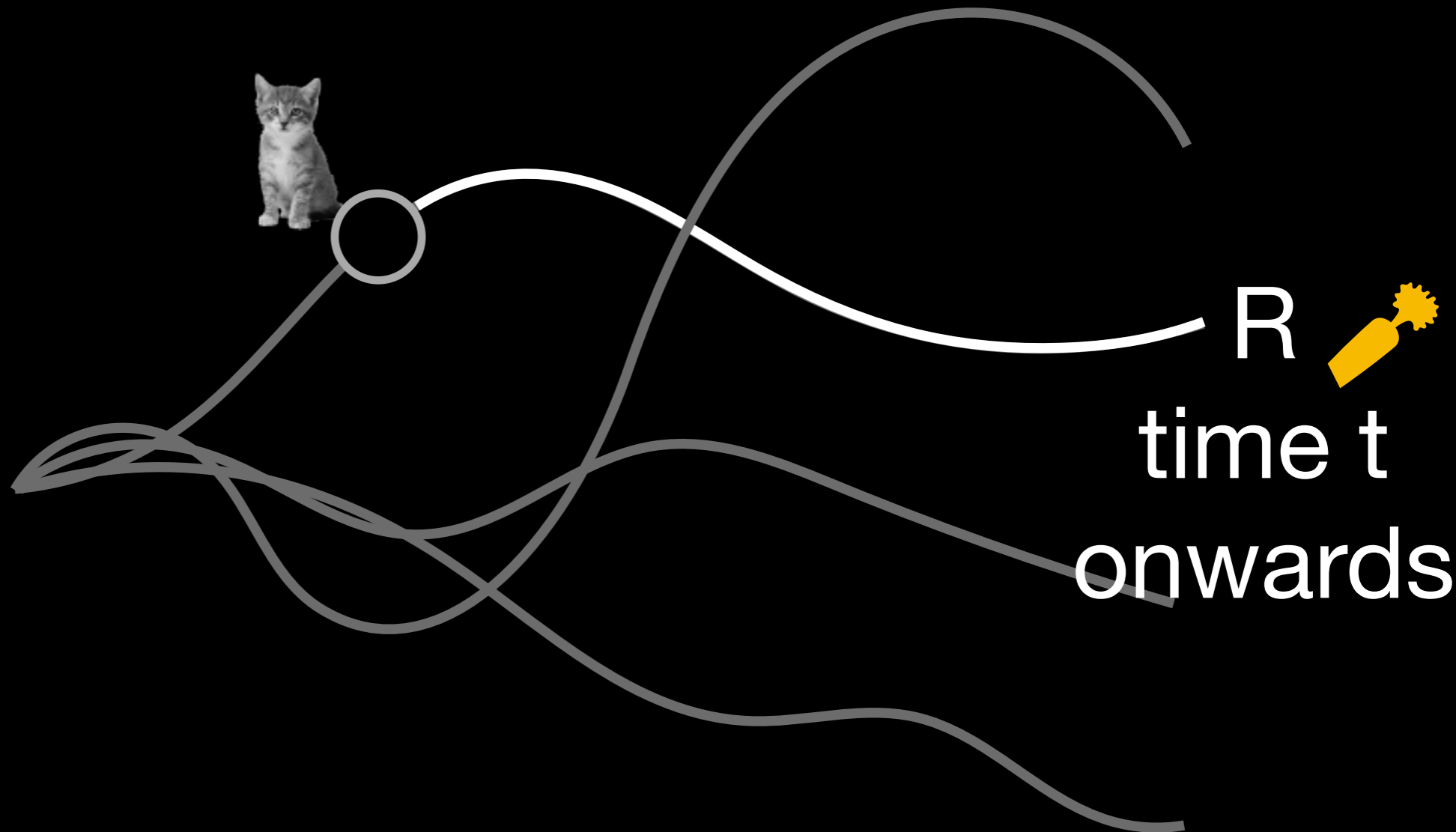
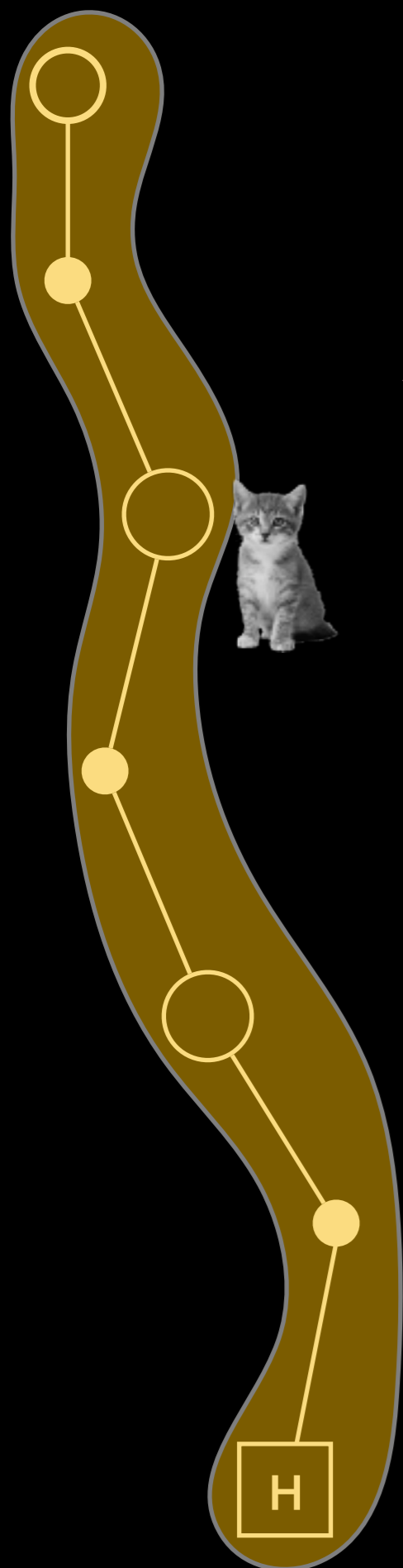
R



Reduce Noise

$\Delta\theta$ to increase $\pi_{\theta}(a_t | s_t) \times$

$R(\tau_{t \text{ onwards}})$



Reduce Noise

$\Delta\theta$ to increase $\pi_{\theta}(a_t | s_t)$ x

$$R(\tau_{t \text{ onwards}}) - b$$



baseline b

(how much is action better than average)



R



R



R



R



$$V = E\{R | s\}$$

Reduce Noise

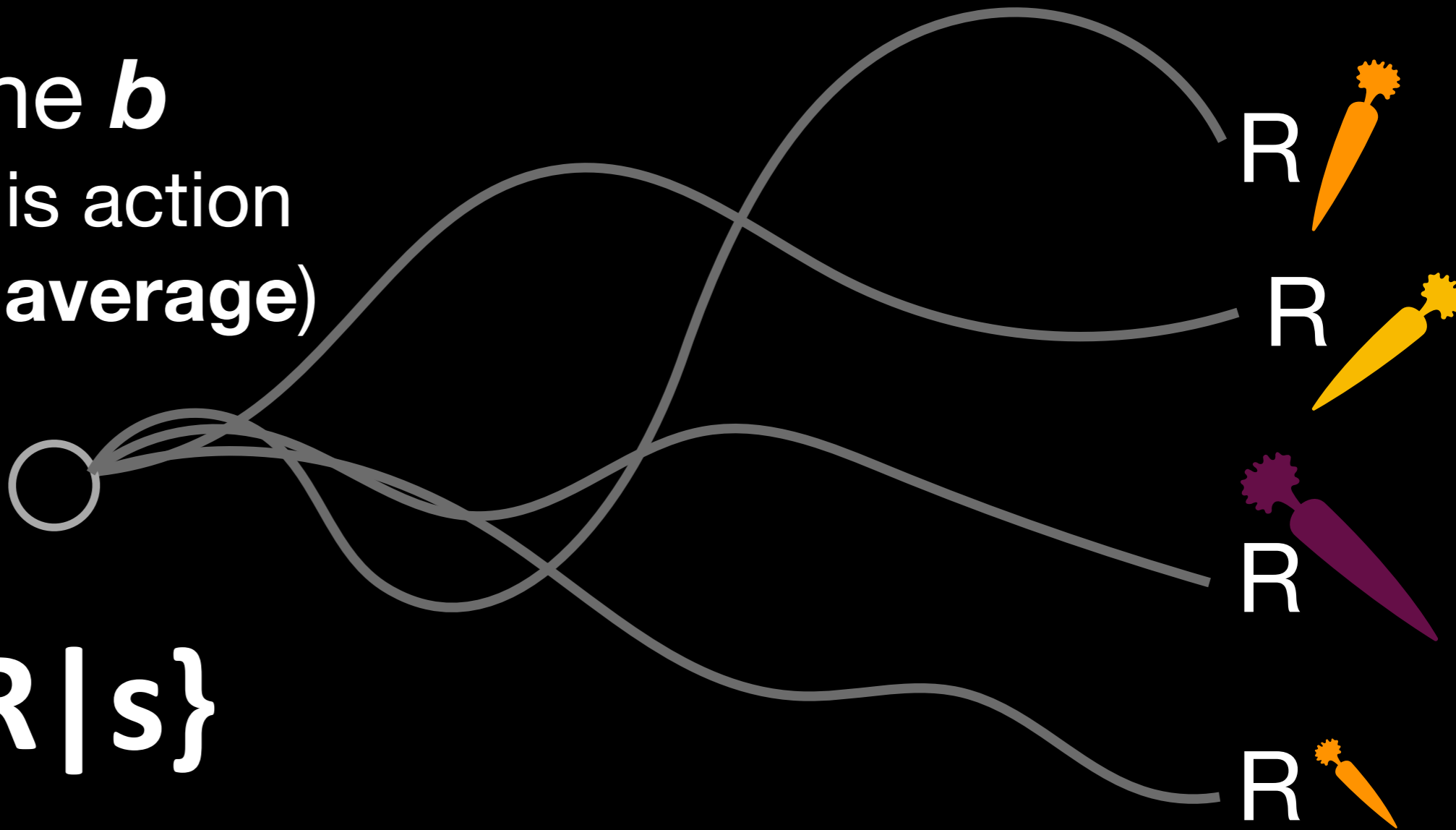
$$R(\tau_{t \text{ onwards}}) - V(s_t)$$

$\Delta\theta$ to increase $\pi_{\theta}(a_t | s_t) \times$



baseline b

(how much is action better than average)



$$V = E\{R | s\}$$

Actor-Critic

Reduce Noise

$$Q(s_t, a_t) - V(s_t)$$

$\Delta\theta$ to increase $\pi_\theta(a_t | s_t)$ x



critic Q
(expected long term
value of action)



$$Q = E\{R | s, a\}$$
$$= E\{r + \gamma V\}$$



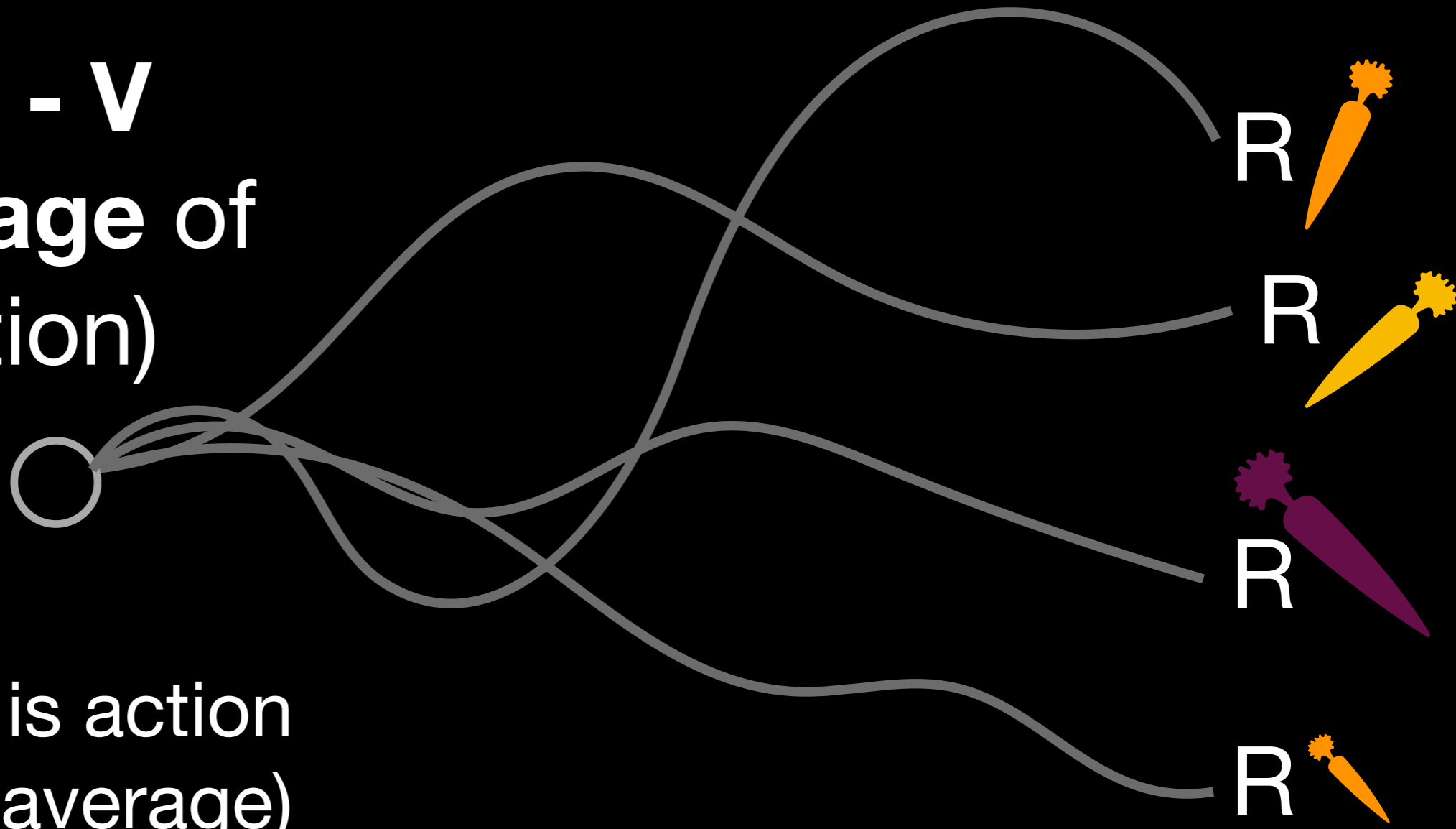
Reduce Noise

$\Delta\theta$ to increase $\pi_\theta(a_t | s_t)$ x

$A(s_t, a_t)$



A = Q - V
(advantage of an action)



(how much is action better than average)

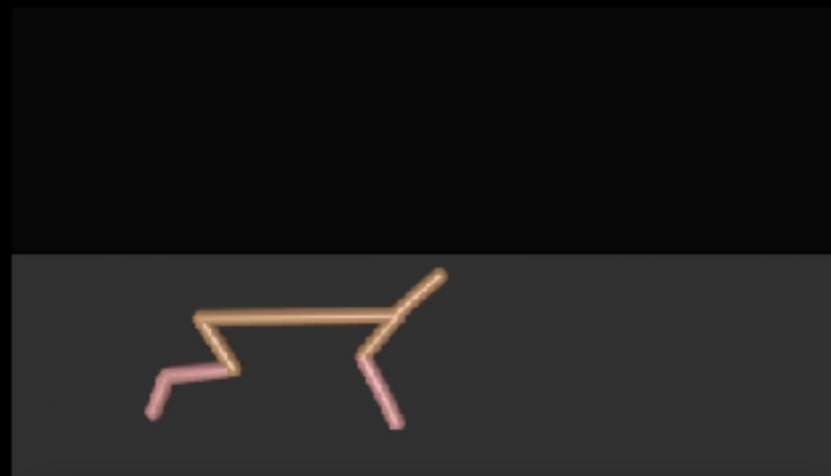
parallelise...

Parallel Asynchronous Training

value and **policy** based methods



<https://youtu.be/0xo1Ldx3L5Q>



<https://youtu.be/Ajjc08-iPx8>



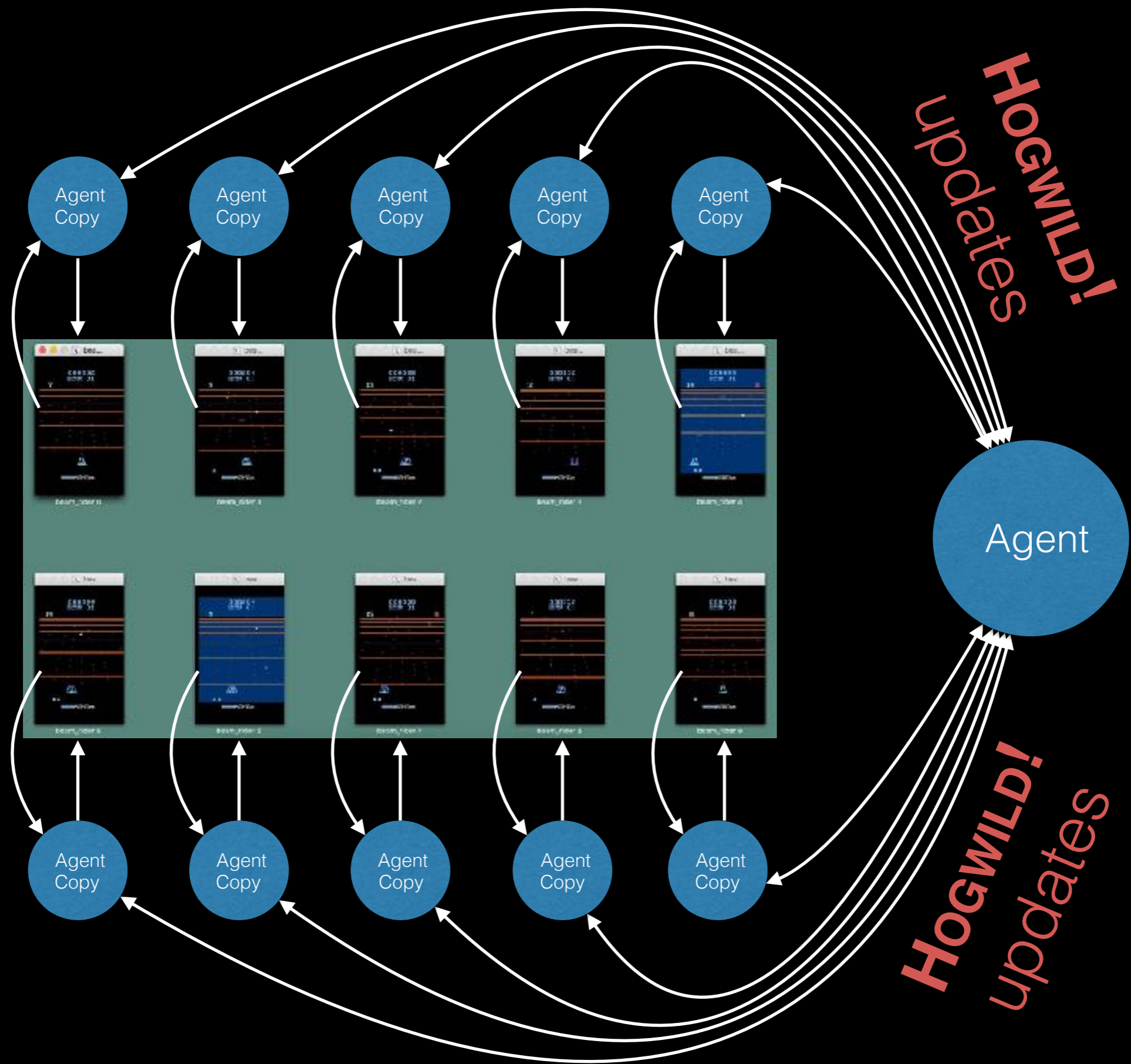
<https://youtu.be/nMR5mjCFZCw>

parallel
agents

shared
parameters

lock-free
updates

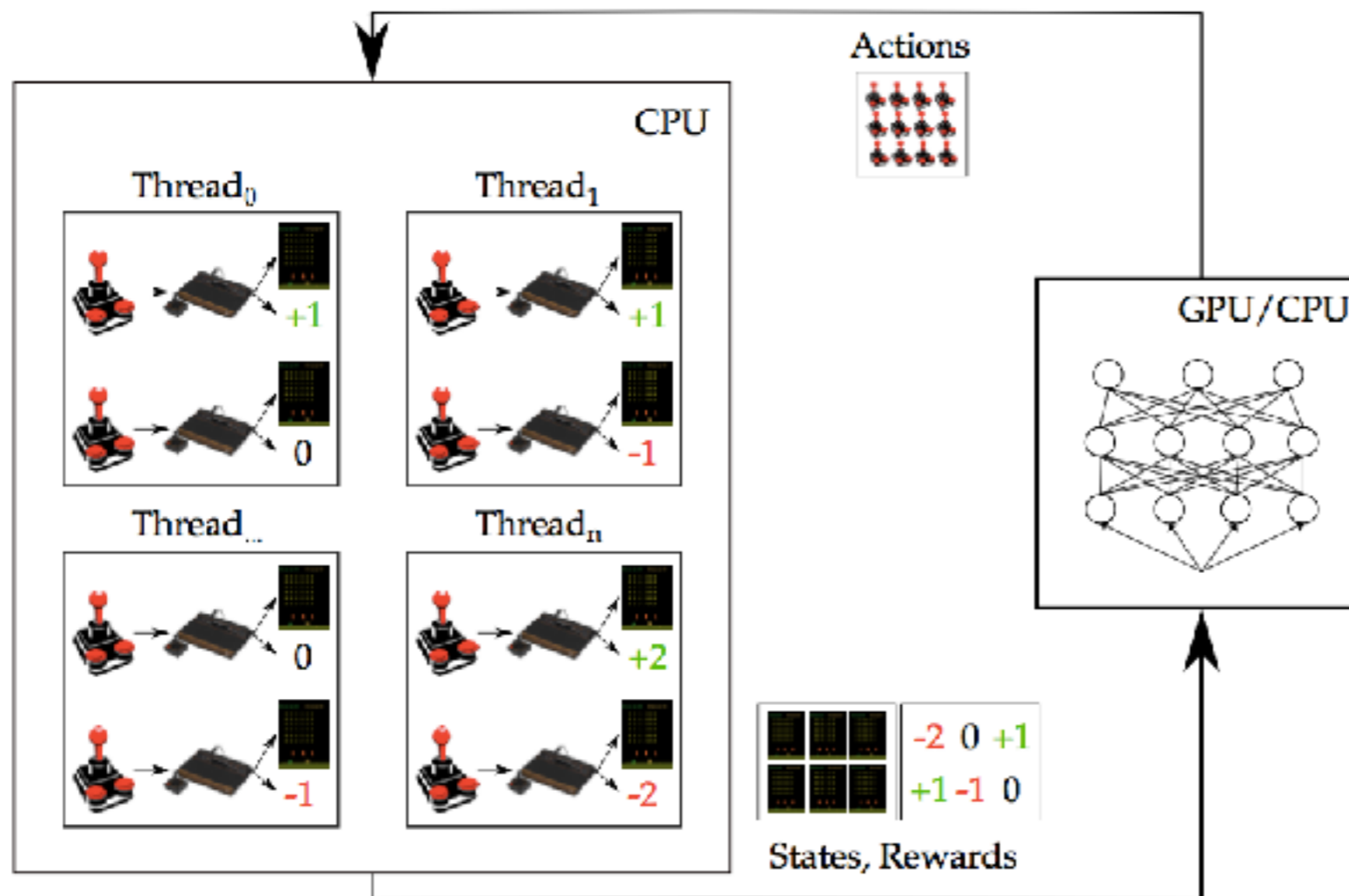
parallel learners



params shared

PAAC

(Parallel Advantage Actor-Critic)



1 GPU/CPU

Reduced
training time

SOTA
performance

<https://github.com/alfredvc/paac>

Efficient Parallel Methods for Deep Reinforcement Learning,
A. V. Clemente, H. N. Castejón, and A. Chandra, **RLDM 2017**



Alfredo
Clemente

code for you to play with...

Rich Sutton's book examples (**exhaustive, must try!**):

<https://github.com/ShangtongZhang/reinforcement-learning-an-introduction>

Telenor's implementation of **asynchronous parallel methods**:

<https://github.com/traai/async-deep-rl>

Alfredo's **faster parallel methods**:

<https://github.com/alfredvc/paac>

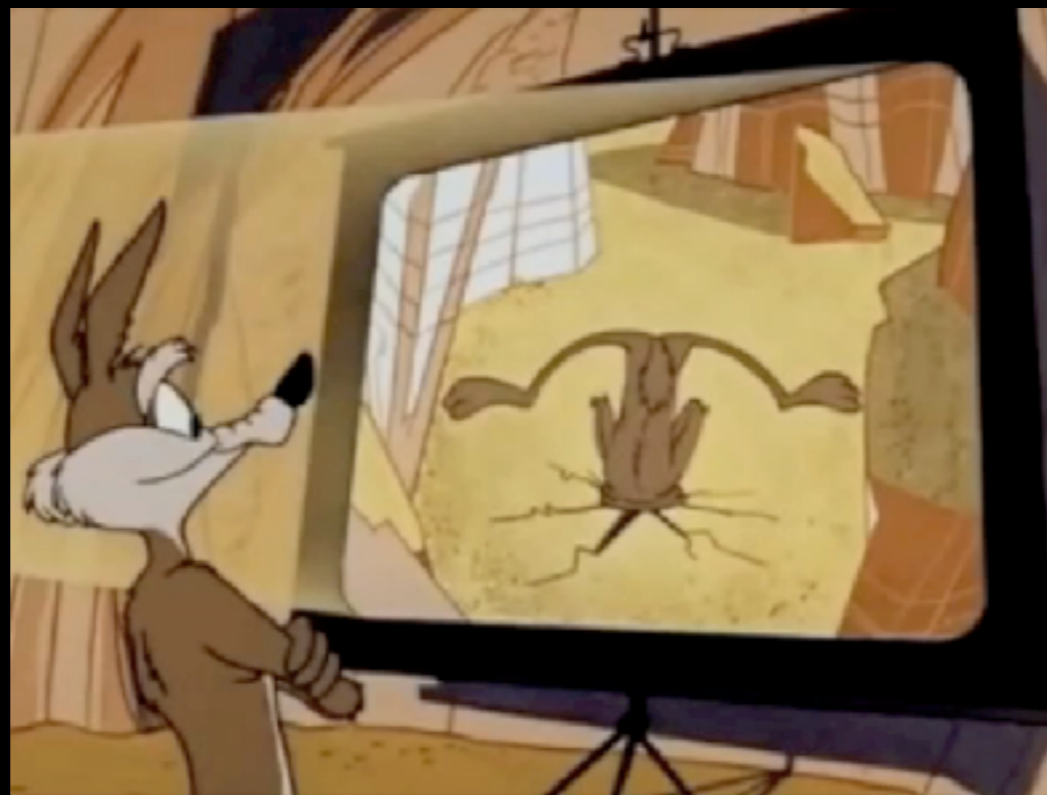
++...



OpenAI Gym BETA



DeepMind Lab



Inspired to
code/apply RL?

Next lecture:
Applications (and some **hacking**)
November 21, 2017

<https://join.slack.com/t/deep-rl-tutorial/signup>